Informal fiscal systems in developing countries*

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Abstract

Governments in developing countries have low fiscal capacity yet face pressures to provide public goods and services, leading them to rely on various unusual fiscal arrangements. We uncover one such arrangement - informal fiscal systems that rely on local bureaucrats to fund the delivery of public goods and services - cataloging its existence in at least 20 countries. Using survey data and government accounts from Pakistan, we show that public officials are expected to cover funding gaps in public services and they do so, at least partially, through extracted bribes. We develop a model of bureaucratic agency to explore when governments benefit from sustaining such systems and investigate their implications for welfare and bureaucrat selection. Informal fiscal systems are more likely to arise when corruption is widespread but public service delivery is relatively easy to monitor. While they provide an effective second-best solution in the presence of moral hazard and adverse selection, they can distort the effective incidence of the tax burden, reduce the incentives of governments to fight corruption, and legitimize bribe-taking. This makes corruption more widespread and thus makes informal systems self-reinforcing.

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1 Introduction

Governments in developing countries have low fiscal capacity (Besley and Persson, 2014), particularly at the local level (Gadenne and Singhal, 2014; Bachas et al., 2021; Dzansi et al., 2022; Balan et al., 2022). These fiscal constraints limit the ability of governments to raise revenues to provide public services. Yet public pressure compels governments in developing countries to attempt to provide these services.[1]

These unique forces have led to the rationing of public goods and services in various developing nations (Banerjee et al., 2007), as well as several unusual fiscal arrangements. For example, governments may rely on the local population to informally deliver public goods (Olken and Singhal, 2011); delegate tax collection to private individuals for profit (Stella, 1993; Coşgel and Miceli, 2009); or even abdicate responsibility to non-state groups (Grossman, 1997; Johnson et al., 1997; Alexeev et al., 2004).

In this paper we uncover the existence of an informal fiscal system: a system in which both taxation and expenditures are managed within the state apparatus but outside its formal fiscal processes. Under the arrangement that we study, state authorities do not provide local public officials with the resources they need to supply public services: too little petrol for police cars, too few materials for flood control. Despite these known shortfalls, authorities also expect local officials to provide these public services, with no guidance on how to raise funds, and no legitimate recourse to raising revenues locally. Officials hence personally fund these public services, with evidence suggesting they rely at least partially on bribes extracted from local communities to do so.

We begin by documenting examples from 18 countries worldwide where bureaucrats are expected to deliver public services without sufficient official funding. We proceed to describe the illustrative case of policing in India, where we conduct an accounting exercise comparing the costs required and the government funds available for patrolling, using survey data from 180 police stations in a large state. We find that the most conservative estimate of the petrol expenditure required for these patrols is more than the amount of funds provided by the government. The funding gap is large relative to the salary of police officers, and evidence suggests that police officials are *"supposed to find other means"*[2] to fill this gap; multiple surveys and reports corroborate corrupt behavior by police.[3]

[1]Developing democracies such as India, Pakistan, Tanzania, and Kenya established universal adult franchise in the 1940s-1950s, at the same time as or earlier than France or Switzerland, and now have larger welfare states than today's rich countries had at historically comparable income levels (Lamba and Subramanian, 2020).

[2]https://www.thehindu.com/news/cities/Hyderabad//article60411103.ece, accessed March 2, 2022.

[3]According to a 2020 Transparency International report, 42% of people in contact with the police in India

Next, we present a more detailed description of an informal fiscal system in a large bureaucracy in Pakistan, in which local (low level) bureaucrats fund public services such as flood control and relief, free food to the public, and the logistics of senior officials' visits to their area. A significant proportion (82%) of the 750 local bureaucrats we surveyed agreed that they provide these services for which they do not receive full official funding and 100% agreed that they contributed to the production of these public services with their own personal funds. Bureaucrats state that this funding represents almost 15% of their monthly income. Altogether, the size of this informal fiscal system is large: approximately 4.3 billion PKR per year, equivalent to 4.5% of the government's main cash transfer program (BISP) in 2015-16 or 558 PKR per eligible family.

We corroborate these survey responses with an independent survey of the bureaucrats' supervisors, who are provincial government agents and mostly belong to the elite Pakistan Administrative Service (PAS). Among the supervisors, 89% confirm that local bureaucrats *fund* the delivery of public services, and 90% of supervisors report that the provincial government does not provide sufficient funds for public service delivery because they expect corruption by the local bureaucrats to cover this shortfall. In contrast, only 27% of supervisors cite the government's inability to raise funds as a reason for the inadequate allocation of resources. This finding aligns with citizen survey reports of frequent bribe payments to local bureaucrats.

Supervisor responses provide important evidence of bureaucrats' involvement in funding public services and the role of corruption in sourcing these funds. While they may have incentives to overstate the provision of public services, they have little reason to highlight their subordinates' involvement in corruption as a justification for the government's reliance on an informal fiscal system. Admitting such corruption could reflect poorly on their managerial oversight or expose them to blame for failing to prevent it. This concern is particularly salient in the survey context, as the government in power at the time had risen to prominence on an anti-corruption platform, resulting in multiple arrests of bureaucrats by the National Accountability Bureau (NAB) on corruption charges.[4]

Additionally, in this context, local officials' wages alone appear insufficient to cover the costs of these services. First, the officials in this context have an average salary of 49,411

had to pay a bribe (https://www.transparency.org/en/publications/gcb-asia-2020, accessed April 30, 2021).

[4]See examples: https://www.thenews.com.pk/print/387720-71-politicians-bureaucrats-being-investigated-by-nab and https://www.thenews.com.pk/print/1136559-nab-s-new-protocol-to-restore-bureaucracy-s-confidence.

PKR, barely higher than the minimum wage in Pakistan of 37,000 PKR. If funding public services were solely drawn from their wages, we would expect high turnover due to the relatively more attractive outside options available to these officials. However, turnover rates remain low. We also show that there is a significant gap (13,000 PKR or 26% of the bureaucrats' monthly wage) between the cost of providing these services, as indicated by supervisors, and the share of salary that bureaucrats report spending on them. We confirm from government accounts that this gap is not due to bureaucrats misreporting their salary and argue that the gap is filled by bribes received by local bureaucrats.

The examples we describe above illustrate a system that is distinct from tax farming, informal taxation, user fees or the provision of public services by non-state actors. Unlike tax farming, bureaucrats are not officially given the right to collect bribes by the government, yet are expected to provide public goods. In informal taxation, local officials only coordinate the voluntary labor or funding provided by citizens, rather than paying for these on their own. Unlike user fees, services for which bribes are paid can differ from the service on which bureaucrats spend the funds in informal fiscal systems: bribes collected for issuing land titles can be used to finance free food to the public. This creates a form of redistribution central to our definition of informal fiscal systems. Finally, in informal fiscal systems, the state itself expects its functionaries to fund public services rather than competing with non-state groups for their provision.

As Acemoglu and Verdier (2000) note, governments choosing to correct market failures through public officials must accept some corruption, since principal-agent problems here are often intractable. However, in our case, the government is actively expecting public officials to provide services without sufficient official funds for them, implicitly acknowledging the existence and use of bribes to fund these services. Why not just tax more, monitor corruption and spend on public goods? What conditions determine whether informal fiscal systems arise?

We develop a model to understand when governments rely on such informal fiscal policies and to investigate their implications for welfare and for the selection of talent in bureaucracies. We study an agency problem between a politician and a bureaucrat. The politician faces pressure from a group of voters to supply public services but only has limited tools to address the moral hazard and adverse selection problems inherent in delegating public service provision to bureaucrats. The bureaucrat is in charge of delivering public services and chooses how much to extract in bribes and what proportion of his income to spend on a public service. Bureaucrats differ according to their honesty (their willingness to accept bribes) and their ability to deliver public services. The politician cannot observe the bureaucrat's type and actions but receives a noisy signal of public service delivery. She draws inferences about the bureaucrat's type based on this signal and decides whether to retain him in the bureaucracy. The desire to be retained creates incentives for the bureaucrat to personally contribute to public services in order to signal his ability. The politician chooses how much formal taxation to raise to finance public services, anticipating that the bureaucrat will also provide personal funding.

In equilibrium, both the amount of public services funded by bureaucrats and the bribes they obtain depend on the quality of information about public service provision and the amount of public services already funded by formal taxes. Decreasing taxes incentivizes bureaucrats to personally fund more services in order to signal their ability. By keeping taxes low, the politician can therefore force dishonest bureaucrats to redistribute the bribes they are taking. However, if taxes are too low, this strategy can also encourage honest bureaucrats, who do not normally take bribes, to start taking bribes in order to fund public services. The politician resolves this trade-off by choosing either an informal policy with low formal taxes but a high level of corruption or a formal policy with no funding from the bureaucrat, higher taxes, and reduced corruption.

Our model offers a way to rationalize the puzzling existence of informal fiscal systems and provides a number of insights into them. We obtain three main results. First, we show that an informal fiscal system is more likely when public service delivery is easy to observe and corruption is widespread (a large share of bureaucrats are willing to take bribes). Under these conditions, it is easier to incentivize dishonest bureaucrats to redistribute bribes than to prevent extraction, which mitigates double taxation (bribes and formal taxes). We further show that even when the share of dishonest bureaucrats is low, sufficiently high observability can sustain an informal system in which honest officials fund services out of pocket to signal ability, which can rationalize some of the examples we document, such as teachers purchasing school supplies with their own funds.

Second, informal fiscal systems can be self-reinforcing. In these systems, public service delivery is financed through bribes. Dishonest bureaucrats, who are more willing to extract bribes, therefore have a financial advantage over honest bureaucrats, and fund more services in equilibrium. Since a higher level of funding serves as a signal of high ability to the politician, dishonest bureaucrats are more likely to be retained in the bureaucracy than honest bureaucrats. Since informal systems are more likely when the share of dishonest bureaucrats is high, informal systems are more likely to be sustained in the future.

Finally, we show that informal fiscal systems can arise as the result of both agency frictions (moral hazard and adverse selection) and political frictions (the unequal representation of different income groups in the political system). When politicians cannot identify dishonest bureaucrats and prevent corruption, informal systems are a valuable second-best option as they can redirect some of the bribes towards public services. When politicians favor a group that bears a large share of formal taxes, informal systems allow politicians to shift the effective tax burden onto other groups and thus become even more likely. However, informal fiscal systems also introduce additional distortions. First, as noted above, they can reinforce the adverse selection of corrupt bureaucrats. Second, as the provision of public services is delegated to the bureaucrats, the level of funding for public services is lower than in formal systems. As a result, social welfare decreases relative to the social optimum (no moral hazard or adverse selection) and the incidence of tax can become more regressive.

The informal fiscal system we uncover has wide-ranging and long-lasting consequences for state capacity development. On the one hand, rents accruing to bureaucrats may be overestimated since some of the bribes are returned as public services. On the other hand, corruption is costly and more distortionary than taxes (Shleifer and Vishny, 1993; Fisman and Svensson, 2007; Banerjee et al., 2012) and the incidence of bribes as a source of funds is different than that of formal taxes. Moreover, informal fiscal systems can reduce the incentives for the government to monitor corruption and legitimize bribe-taking for the bureaucrats thus serving as a gateway to more corruption. In fact, supervisors of local bureaucrats in Pakistan indicated that these officials were happy to provide the public services precisely because they saw it as a way to justify collecting bribes.

Our paper contributes to the literature on public finance in developing countries. Broadly, it helps in understanding why developing countries consistently fail to both raise revenues (Gadenne and Singhal, 2014) and to invest in fiscal capacity (Acemoglu et al., 2005; Besley and Persson, 2009, 2010, 2014; Besley et al., 2013). Our work also adds to studies documenting that information frictions are an important determinant of how governments collect taxes (Kiser, 1994; Balan et al., 2022). Narrowly, our paper contributes to the literature on informal taxation (Olken and Singhal, 2011; Gadenne and Singhal, 2014; Jack and Recalde, 2015; Lust and Rakner, 2018; Van den Boogaard et al., 2019) by exploring a new form of informal fiscal policy. In particular, we explore the possibility that decentralized public good provision relies on direct payments from the local bureaucrats (potentially through the redistribution of bribes), rather than on voluntary contributions from the local population. Another strand of this literature emphasizes the role of political accountability in determining "bureaucratic overload" (Dasgupta and Kapur, 2020), where bureaucrats are expected to complete tasks for which they do not have sufficient resources. We complement these findings by showing that governments can expect bureaucrats to use bribes to cover the gap in official funds and hence, the lack of resources might be overestimated.

Our findings also contribute to three strands of the literature on corruption. First, we describe a new force that can explain the persistence of corruption (Tirole, 1996; Dutta et al., 2013). Corruption can persist because it allows the government to fund public services with low levels of formal taxes and because corrupt bureaucrats can outperform honest bureaucrats in delivering public services. Second, redistribution of bribes through informal fiscal systems makes the welfare calculations related to corruption ambiguous (Shleifer and Vishny, 1993). Third, we explore a new facet of the relationship between corruption and bureaucrats' incentives (Tirole, 1986; Mookherjee and Png, 1995; Niehaus and Sukhtankar, 2013; Sanchez De La Sierra et al., 2024), showing that governments can affect corruption by choosing the level of taxes.[5] The theoretical study most closely related to our work is Besley and McLaren (1993), who show that governments may deliberately set low wages on the assumption that bureaucrats will supplement their income through bribes. Our paper extends this framework by examining more general forms of informal fiscal systems where bureaucrats personally fund public goods rather than just their own consumption. In our model, the government strategically chooses formal tax levels rather than wages to incentivize this redistribution. Consequently, corruption acts not merely as a compensation mechanism but as a fiscal tool that alters the effective tax burden and distorts the selection of bureaucratic talent.

2 Motivating examples

Situations in which state officials are expected to fund public services out of their own pockets are common around the world. Public school teachers even in developed countries like the USA often pay for school supplies.[6] The underlying funds can be provided by parents or the community (e.g. bake sales) or can come out of the teachers' pockets.

[5]See Becker and Stigler (1974); Di Tella and Schargrodsky (2003); Olken (2007); Reinikka and Svensson (2011); Corbacho et al. (2016); Debnath et al. (2023) for some of the tools already studied in the literature.

[6]See, e.g., https://www.theguardian.com/us-news/2021/dec/13/teachers-scramble-dollar-bills-south-dakota-dash-for-cash, accessed April 8, 2022.

In developing countries, the source of funds can be more controversial. In the Democratic Republic of Congo, former President Mobutu Sese Seko told the police and army "débrouillez-vous" (live off the land), thereby acknowledging bribe taking as a substitute for salaries (Weigel and Kabue Ngindu, 2023). Prud'Homme (1992) also describes how wages for local officials in the Democratic Republic of Congo are deliberately kept very low by the government who expected officials to fund themselves through other means such as collecting bribes. In this case too, the public good of law and order is expected to be funded by the civil servants. These arrangements parallel the concept of "capitulation wages" discussed in Besley and McLaren (1993).

More broadly, an online search of local media brought up 18 different countries in which similar instances were reported. In seven of those examples, bribes are reportedly used to cover shortfalls in public funding, while in six of those the shortfall is covered by the bureaucrats' own wages (in the remaining cases, the source of funds is unclear from the article). These countries span diverse regions, including Africa, Latin America, Southeast Asia, Central Asia, Eastern Europe, and the Middle East, illustrating the widespread prevalence of situations where bureaucrats are expected to fund public service provision in the absence of official resources. Table 1 lists these examples.

In India, we document a similar system in the police force. The fact that public service providers in India suffer from severe resource constraints is well-documented (Kapur, 2020). We carried out a careful accounting exercise for monthly petrol costs incurred at police stations. In 2018, we surveyed a representative sample of the Station House Officer (head of the police station) in each of 180 police stations with a jurisdiction covering nearly 24 million people in a large state in India. The survey gathers details on the number and type (car or motorcycle) of police vehicles, the average number of kilometers traveled, as well as the monthly budget received for "Petrol, oil and lubricants". We combine the data on the type of vehicle, the car dealer-reported mileage provided by these vehicles, and the average number of kilometers traveled to generate the number of liters of petrol needed.[7] Using the minimum price per liter of petrol in the survey month, we generate an (extremely conservative) estimate of the required petrol budget.

Comparing the budget required with the reported budget received, we find that the

[7]While it is possible that the average number of kilometers are overestimates, note that these are provided by the station head, based on knowledge of patrol beats and official travel, while police officers lower down the hierarchy are expected to fund this travel. This makes inflated numbers unlikely; moreover, even if the station head somewhat overestimates the average distance traveled, the gap to officially provided funding is substantial.

average station experiences a monthly shortfall of 14,845 INR (representing 95% of our estimate of expenditure, see Table A1). Not even a single station reports having enough funding to do regular policing patrols, even with these conservative assumptions; less conservative assumptions result in an average shortfall of 15,256 INR (Table A2). Official budget figures for "Petrol, oil, and lubricants" funds allocated to police stations corroborate the survey data, with a shortfall of 8,768 INR even assuming zero leakage.[8]

How, then, do the police cover these deficits? Newspaper reports and informal interviews with both senior and junior officials by the authors reveal that junior officers are "supposed to find other means" to support fuel budget shortages.[9] Some survey respondents reported that they have to use their personal vehicles for on-duty responsibilities; many others might have to resort to extracting bribes. It is then no surprise that according to a nationally representative survey by Transparency International in 2019-20, 42% of people in India who had contact with the police in the previous twelve months paid a bribe, nearly twice the average rate in Asia, and the highest of all public services in India (Asia Global Corruption Barometer).

3 Informal fiscal system in Pakistan

We now document the existence of an informal fiscal system in Pakistan through surveys of bureaucrats. We use data from three sources: 1) a telephone survey of a random sample of 750 local bureaucrats out of a total of 6209 across Punjab in 2020; 2) a telephone survey of 35 direct managers of these local bureaucrats (stratified on districts, randomly sampled 42 of 141) in 2020; and 3) a citizen survey carried out by a private firm for the provincial government in 2009, explicitly surveying individuals that have interacted with the local bureaucrats (comprising 1,402 men that either own or rent land).[10]

[8]These calculations are consistent with the large number of news reports on the lack of funds for petrol across India: see, for example the case of Mumbai `https://www.dnaindia.com/mumbai/report-mumbai-cops-inadequate-fuel-for-patrol-vehicles-2781055`, accessed June 17, 2021.

[9]See for e.g. `https://www.thehindu.com/news/cities/Hyderabad/new-police-vehicles-are-welcome-what-about-fuel/article6146002.ece`, accessed June 17, 2021. Separately, in an interview with one of the authors, an Additional Director General of Police pointed out that women are much less likely to make it to SHO of the station precisely because they are unable to raise the funds required for things like officials visits, petrol, etc. Finally, such examples are so common they even make it into movies: in *Santosh*, the UK's official entry to the 2024 Oscars, a young female constable in India (inadvertently) gets a bribe and then is asked to take a body of a murder victim to the morgue, for which she is forced to hire a private mini-truck and pay for it literally using the same cash from the bribe.

[10]The questions for local bureaucrats used here were part of a broader survey of their career background and traits but the survey of managers was carried out specifically for this paper.

3.1 Private funding of public services by local bureaucrats

We first examine the extent to which bureaucrats participate in providing underfunded local goods and services, the sources of funds for this provision, and the share of income bureaucrats spend (Table 2). Eighty-two percent of local bureaucrats report providing public goods and services outside of their formal budget. Supervisors corroborate the bureaucrats' involvement (98%). All local bureaucrats (100%) and 89% of supervisors agree that local bureaucrats supply funds for these services.

Our data also indicates that this funding is not trivial. Bureaucrats note that they spent 7,412 PKR a month - 15% of their average monthly income of 49,411 PKR - on delivering public services. The total size of this informal fiscal system is significant – around 4.3 billion PKR per year,[11] equivalent to 4.5% of the government's main cash transfer program (BISP) in 2015-16.[12] This amount can underestimate their overall rupee contribution as the bureaucrat's total income can be larger if they receive money from other sources such as bribes.

Finally, these funds are not simply prepayments from the bureaucrats that the state reimburses. Only 8% of supervisors agree that field bureaucrats file to be reimbursed for these expenses.

In Table 3, we further investigate three commonly funded goods and services: 61% of bureaucrats agree that they provide flood control and relief, 25% provide free food to the public, and 82% arrange logistics during official visits. Again, supervisors confirm that bureaucrats' provide these three services, with 90% or more agreeing.

Meanwhile, the extent to which bureaucrats are financially involved differs by type of service. While bureaucrats report contributing a majority of the funds in both the provision of free food and the organization of officer visits, they contribute a larger portion for official visits. Supervisors believe that the proportion of funds covered by bureaucrats is lower but still significant. For flood control and relief, they believe that the government contributes 73% while bureaucrats bear 13% of the costs. In the case of provision of free

[11]Using the supervisor survey, we estimate that the total costs per Tehsil of public services borne by local bureaucrats is PKR 886,757 per month. Given an average of 44 officials in each Tehsil, the spending amounts to PKR 20,154 per official per month. We used the supervisor survey for these estimates as they have less incentives to misreport the costs and because the data on costs of flood control is missing in the bureaucrat survey. To arrive at the figure for the total size of the informal fiscal system we used the amount spent per official in a tehsil per month of 20,154 PKR and multiplied it by 12 months and 44 bureaucrats per Tehsil in 404 Tehsils in Pakistan.

[12]https://bisp.gov.pk/Detail/Zjk1OWZkYzEtZWE2Yy00NThlLThhZDAtMzc4MWM1OWIyZjU4

food for the public, they report that local philanthropists bear the largest burden (73%) while bureaucrats fund 15% of the costs and the government only 11%.

The existence of such practices raises two questions: why do bureaucrats agree to provide these funds and do these funds come exclusively out of their official wages?

3.2 Bureaucrats' motivations

Bureaucrats indicate two main reasons for agreeing to pay for these services: pressure from colleagues and altruism. Table 4 shows that 62% of officials are willing to fund the provision of the public services due to social pressure from colleagues while 30% cite altruism towards citizens as a reason. Supervisors believe that self-interest rather than altruism plays a bigger role than bureaucrats want to admit: 76% of supervisors think that officials are willing to spend out of their pocket due to career concerns, while only 20% cite social pressure and none of them mention altruism. Moreover, 39% of the supervisors, who themselves volunteered this response under the category "Other", believe that the officials are content to maintain this informal fiscal arrangement because it enables them to continue to engage in corruption.

We can relate these motivations to the heterogeneity in the source of funds across different types of services. If bureaucrats are motivated by social pressure or career concerns, then they should be more likely to provide services that are easier to observe for their colleagues or supervisors. For instance, supervisors can directly observe the success of senior officials' visits. By contrast, assessing whether the correct flood control measures were implemented is more difficult.[13] In Section 4, we show how the observability of the service provision can affect the incentives of the bureaucrat and the likelihood of an informal fiscal system.

3.3 Sources of funds used by bureaucrats

While our data reveals that bureaucrats finance local public goods from their own funds, rather than official government funding, these funds could either come from the bureaucrats' personal wages or from bribes.

[13]The heterogeneity across services is less consistent with the altruism motivation: altruistic bureaucrats would be more involved in activities that help citizens directly such as flood control or food provision than official visits.

While plausible, it seems unlikely that the funds used for public services come exclusively from the bureaucrats' official wages. The officials in this context are not part of an elite civil service and their average salary (PKR 49,411) is relatively low. The funding could account for up to 40% of their income. If bureaucrats only spent out of their own pockets, their net annual salary would drop below the minimum wage of PKR 37,000; at this salary, their outside options would be dominant. Yet, we do not see these bureaucrats leaving their jobs in droves, indicating that they obtain funding from other sources.

We present three pieces of evidence that suggest that bribes extracted from the local population could be a key source of funding: (1) results from the supervisor survey, (2) an accounting exercise comparing the salary of the bureaucrat with the cost of providing the public services and (3) results from a citizen survey.

Table 2 Panel B shows that 90% of the supervisors believe that the government does not fully fund services as it knows that the local bureaucrats earn bribes. Only 27% think that the shortfall in funds is due to difficulty in raising money through taxes and borrowing by the government. Supervisors also emphasize that a significant cost of such an informal fiscal system is the perpetuation of corruption. Specifically, as noted earlier, 39% of supervisors volunteered the view that local bureaucrats are willing to spend out of pocket because it reduces the likelihood of being held accountable in the future. The government's expectation of funding public services provides local officials with a justification for engaging in bribery.

Supervisor responses constitute an important piece of evidence that the funding gap is filled through corruption. Supervisors had little incentive to openly report that their subordinates are involved in corruption. Acknowledging this reflects badly on their management skills or puts them at risk of being blamed for not preventing this corruption. As discussed earlier, this issue was particularly sensitive in the context of the survey, conducted under a government that prioritized anti-corruption measures and actively pursued bureaucrats through the National Accountability Bureau (NAB).

We supplement supervisors' responses with a back-of-the-envelope calculation: we calculate the share of the costs of these activities that are borne by local bureaucrats, as reported by supervisors, and compare these costs with the share of *official* income that they claim to spend on these activities. The funding required is 20,154 PKR per official per month. This is much higher than the 7,415 PKR per official per month that the bureaucrats report spending out of their official income.

This funding gap of approximately PKR 13,000 (PKR 20,154 minus 7,415) can be due to either bureaucrats misreporting the size of their official salary or the fraction of their expenditure that bureaucrats report spending on these services. We corroborated the average salary of these bureaucrats from the AGPR, the government body responsible for paying salaries, and did not find a discrepancy. Moreover, surveyor demand effects would likely push bureaucrats to report a larger - rather than smaller - fraction of their expenditure spent for providing services, suggesting that the size of the gap is potentially an underestimate.

Finally, a citizen survey corroborates the payment of bribes to these local bureaucrats (Table A3). Sixty-five percent of citizens report that services are denied to them unless they make unofficial payments to these local officials and 82% state that they pay bribes to overcome difficulties in accessing services.

This evidence, along with the previously discussed cases, suggests that bribes can explain part of the gap between the costs of funding public services and the amount provided by the government. This provides the basis for an informal fiscal system. The government appears to be aware of the corruption by local bureaucrats, and expects them to pay for public goods and services in return. In turn, these bureaucrats appear to support this system because it allows them to engage in corruption with reduced accountability. In the following section we present a simple theoretical framework to investigate the conditions that determine whether informal fiscal systems arise and their implications for welfare and bureaucrats selection.

4 Model

We consider a politician and a bureaucrat interacting over two periods. The politician faces pressure from a homogeneous group of voters to provide public services while keeping corruption and taxes low. The bureaucrat is in charge of delivering public services, which he can chose to fund out of his own pocket, and can extract bribes from voters. The politician faces both adverse selection and moral hazard: she cannot observe the bureaucrat's type, and bribes and personal funding are not contractible. The only way the politician can affect the amount of public services and the bureaucrat's behavior is by choosing the level of taxes. We want to understand what tax level the politician chooses in equilibrium and the resulting amount of public services, private funding, and bribes.

The bureaucrat's type varies across two dimensions. A bureaucrat can be low ($\omega = 0$) or high ability ($\omega = 1$) and can be either honest ($\theta = H$) or dishonest ($\theta = D$). The bureaucrat's honesty is known to the bureaucrat but not to the politician who believes the bureaucrat is dishonest with probability $\nu = \mathbb{P}(\theta = D)$. The bureaucrat's ability is unknown to both players who share a prior that the bureaucrat's ability is high with probability $\mu = \mathbb{P}(\omega = 1)$.[14] Honesty and ability are independently distributed.

In each period, the politician moves first and chooses a lump-sum tax $\tau \in [0, +\infty)$. The bureaucrat is responsible for delivering public services. After observing τ, he chooses how much to extract in bribes $b \in [0, +\infty)$ and what amount of public services to privately fund, denoted e. The bureaucrat cannot spend more on public services than his total income, which equals his exogenously-given wage, w, plus the bribes he obtains: $0 \leq e \leq w + b$. The total amount of public services provided is $y = \omega(\tau + e)$. Taxes and personal funding by the bureaucrat are substitutes to produce public services, but public services are only delivered if the bureaucrat is of high ability ($\omega = 1$).[15]

The politician cannot observe bribe-taking nor the amount of private funding and can only imperfectly observe whether the bureaucrat delivered the public services. These information frictions can create an agency problem and constrain the politician's ability to implement her preferred level of public service. To model these information frictions, we assume that the population needs a level $\bar{y}$ of public services that is not perfectly observed by the politician nor the bureaucrat.[16] Both players share the prior belief that the level of public services needed is distributed according to some CDF F, $\bar{y} \sim F$, where F is strictly increasing over some interval $[0, \Psi]$, is differentiable, strictly concave on $[0, \Psi]$, and such that $F(0) = 0$ and $F(\Psi) = 1$. Let f denote the derivative of F which we assume is continuous on $(0, \Psi)$. At the end of the first period, the politician observes an imperfect signal $s \in \{0, 1\}$ indicating whether the needs of the population have been met. If the needs have not been met, $y < \bar{y}$, the politician receives signal $s = 0$. If the needs have been met, the politician receives signal $s = 1$ but only with some probability $\phi \in (0, 1)$, and receives $s = 0$ otherwise. That is, the signal realization $s = 1$ perfectly reveals that the needs have been met, but the realization $s = 0$ only imperfectly reveals whether the needs have been met. Given this signal, the politician updates her beliefs about the type of the

[14]Symmetric uncertainty is a standard assumption of career concern models, see e.g. Holmström (1999). In the context we study, bureaucrats could be unaware of how efficient they are at using funds (i.e., how little funds they waste when providing a service) until they gain more experience.

[15]The results would continue to hold as long as the low-ability bureaucrat delivers the public services with a lower probability than the high ability-bureaucrat.

[16]For instance, the players might not be able to perfectly assess the severity of a flood.

bureaucrat and decides whether to retain the bureaucrat for the second period. Let $r = 1$ denote the decision to retain the bureaucrat. If the politician does not want to retain the bureaucrat, she can transfer him into another service or district and replace him by a new bureaucrat randomly drawn from a pool. Let $r = 0$ the decision to replace the bureaucrat.

The politician's objective is to maximize the intertemporal sum of utilities of a subset of voters over the two periods. We normalize the discount factor to 1. In each period, these voters receive a payoff of $\lambda \in (0, +\infty)$ if the level of public services meets their needs ($y_t \geq \bar{y}$). The voters pay taxes τ and each unit of bribe b imposes a cost η on them, where $\eta > 1$ captures the distortionary cost of bribes. The voters' per-period utility is therefore:

$$v_t(y_t, \tau_t, b_t) = \begin{cases} \lambda - \tau_t - \eta b_t & \text{if } y_t \geq \bar{y} \\ -\tau_t - \eta b_t & \text{if } y_t < \bar{y} \end{cases}$$

In each period, the bureaucrat gets a base wage w_t and the bribe he extracts b_t minus the amount he redistributes e_t. The bureaucrat's wage is exogenously given, can vary across the two periods, and is not part of the politician's utility. In addition, the bureaucrat faces a cost of extracting bribes, $C(b_t, \theta)$, which can capture the moral cost of corruption, the bureaucrat's bargaining power against citizens, or the risk of getting caught and punished. The function $C(b, \theta)$ is strictly increasing, continuously differentiable, and strictly convex in b. Let $c(b, \theta)$ denote the partial derivative of $C(b, \theta)$ with respect to b. A key feature is that the marginal cost of taking bribes is higher for the honest type than for the dishonest type: $c(b, H) > c(b, D)$, $\forall b \in [0, +\infty)$. We normalize the honest type's marginal cost of taking bribes at $b = 0$ to $c(0, H) = 1$. This implies that an honest type does not take bribes for his own consumption since his direct payoff from taking bribes, $b - C(b, H)$, is decreasing in b for any $b \geq 0$.[17] However, as we show below, the honest type might still want to take bribes to fund public services if his incentives to do so are sufficiently strong. We normalize the payoff of a bureaucrat who is not retained to zero. The bureaucrat's per-period payoff is therefore:

$$u_t(e_t, b_t \mid \theta) = w_t + b_t - e_t - C(b_t, \theta)$$

To summarize, the timing is as follows. In the first period,

[17]Since C is strictly convex, $c(b, H) > c(0, H) = 1$ for any $b \geq 0$, so the derivative of $b - C(b, H)$, $1 - c(b, H)$ is negative.

1. The bureaucrat privately learns his honesty θ.

2. The politician chooses the tax level τ_1.

3. The bureaucrat observes τ_1 and chooses funding e_1 and bribes b_1.

4. The politician observes the signal s and decides whether to retain the bureaucrat or replace him with a randomly-drawn bureaucrat.

In the second period,

1. The politician chooses τ_2.

2. The bureaucrat observes τ_2 and chooses e_2 and b_2.

3. The game ends.

Equilibrium concept. We solve for the weak perfect Bayesian equilibrium in pure strategy. In the first period, the politician's strategy is a tax $\tau \in [0, +\infty)$ and the bureaucrat's strategy is a choice of bribe and private funding as a function of his honesty and the politician's choice of tax: $(b, e) : \{H, D\} \times [0, +\infty) \to [0, +\infty) \times [0, w + b]$. At the end of the first period, the politician updates her beliefs about the type of the bureaucrat according to Bayes rule, given the signal s and her conjecture of the bureaucrat's equilibrium choice of bribe and funding. The politician's retention strategy is a function mapping the signal s into a decision to retain the bureaucrat or not: $r : \{0, 1\} \to \{0, 1\}$. The politician's second period strategy is a choice of tax rate given her beliefs about the bureaucrat's type. If retained, the bureaucrat updates her beliefs about her own ability according to Bayes rule and chooses a second period level of bribes and private funding. If the politician is indifferent between several level of taxes, we assume that she chooses the highest level.[18]

5 Analysis

We begin by solving for the second-period decisions of the bureaucrat and the politician. We then solve for the politician's decision to retain the bureaucrat or not at the end of the first period given the information she obtains about the provision of public services. Finally, we solve for the bureaucrat's first-period action given this retention rule and the politician's choice of tax in the first period. All proofs are provided in the appendix.

[18]This is simply a tie-breaking rule for the knife-edge cases where parameters are such that there are several maxima.

5.1 Second period actions and politician's decision to retain the bureaucrat

The politician's decision to retain the bureaucrat depends on her expected second-period payoff from different types of bureaucrats. Her expected payoff, in turn, depends on her belief about the different types of bureaucrats following the signal she receives about the bureaucrat's first-period performance. To focus on the main trade-offs faced by the politician, we assume that there are no opportunities for corruption in the second period so that $b_2^* = 0$ for all types $\theta \in \{H, D\}$. This assumption has two implications. First, the politician only cares about retaining high ability bureaucrats, independently of their honesty. Second, honest and dishonest bureaucrats have the same expected benefits of being retained in the second period. We discuss these implications in Section 5.6.

In the second period, the bureaucrat has no incentives to privately fund services since the game ends so $e_2^* = 0$ for all types $\theta \in \{H, D\}$ and any history of actions. Given the anticipated lack of funding, the politician chooses a level of tax τ_2 that depends on her beliefs about the bureaucrat's ability since there is a possibility that the taxes are wasted by a low-ability bureaucrat. Specifically, the politician chooses a tax $\tau_2^*(r = 1)$ which maximizes $\mathbb{P}(\omega = 1 \mid s)\lambda F(\tau) - \tau$ if she retains the bureaucrat and a tax $\tau_2^*(r = 0)$ which maximizes $\mu\lambda F(\tau) - \tau$ if she does not. Given this expected second-period behavior, the politician uses the following retention rule:

Lemma 1. *The politician retains the bureaucrat if and only if $s = 1$.*

The politician's second-period payoff from retaining the bureaucrat is higher than her payoff from replacing him if the bureaucrat is sufficiently likely to have a high ability. Since the first-period public service provision depends on ability, as $y = \omega(e + \tau)$, the politician is more likely to receive signal $s = 1$ when the bureaucrat is high ability and guaranteed to receive signal $s = 0$ when the bureaucrat is low ability. As a result, signal $s = 1$ perfectly reveals the bureaucrat to be high ability while signal $s = 0$ indicates that the bureaucrat is more likely to be low ability than a randomly-selected bureaucrat.

5.2 Bureaucrat's first-period strategy

We can now turn to the bureaucrat's first-period choice of bribe and private funding of public services. Throughout this subsection, we omit the period t subscripts for taxes, bribes, and funding to ease the notation, but keep the subscripts on the wages. Given the politician's retention rule from Lemma 1, the probability of being retained in the second period is $\mathbb{P}(s = 1) = \phi\mathbb{E}_\omega[\mathbb{P}(\omega(\tau + e) \geq \bar{y})] = \phi\mu F(\tau + e)$. Since the bureaucrat takes no bribe and provides no funding in the second period, the payoff of being retained is simply w_2. Given some tax τ, the bureaucrat's choice of b and e therefore solves:

$$\max_{b,e} \quad w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b, \theta) \quad \text{s.t.} \quad 0 \leq e \leq w_1 + b,\ 0 \leq b$$

The level of bribes, b, depends on the honesty of the bureaucrat and on the budget constraint. If the budget constraint does not bind ($e^*_\theta(\tau) < w_1 + b^*_\theta(\tau)$), the choice of bribe is independent of the decision to privately fund public services. In this case, the honest type does not take any bribes since $c(b, H) \geq 1$ for any $b \geq 0$, while the dishonest type sets the marginal benefit of taking bribes equal to its marginal cost: $1 = c(b, D)$. If the budget constraint binds ($e^*_\theta(\tau) = w_1 + b^*_\theta(\tau)$), taking bribes loosens the budget constraint and therefore allows the bureaucrat to increase his probability of retention. As a result, when the constraint binds, the level of bribes, b, depends on the probability and value of retention ($\mu\phi w_2 F(\tau + e)$).

The level of private funding, e, also depends on the bureaucrat's honesty and whether the budget constraint binds. When the budget constraint does not bind, the bureaucrat simply sets the marginal benefit of additional funding (increasing the retention probability) equal to the marginal cost: $\mu\phi w_2 f(\tau + e) = 1$. This funding is therefore independent of the bureaucrat's honesty. When the budget constraint binds, the marginal cost of increasing funding is the marginal cost of taking additional bribes, so the optimal level of funding solves $\mu\phi w_2 f(\tau + e) = c(e - w_1, \theta)$ and the funding depends on the bureaucrat's type.

Finally, note that a higher tax level decreases the marginal benefit of personal funding since F is concave. The tax level therefore determines whether bureaucrats want to fund public services at all and whether their budget constraint is binding. In particular, there exist three thresholds, denoted τ_1, τ_2 and τ_3, that determine whether and how the bureaucrat provides funding.[19] The following Lemma characterizes the bureaucrat's funding and bribe taking behavior. We say that a bureaucrat takes *additional bribes* if he takes more bribe to fund public services than he would without providing private funding.

Lemma 2.

- *If $\tau < \tau_1$, both types privately fund public services and take additional bribes to fund them.*

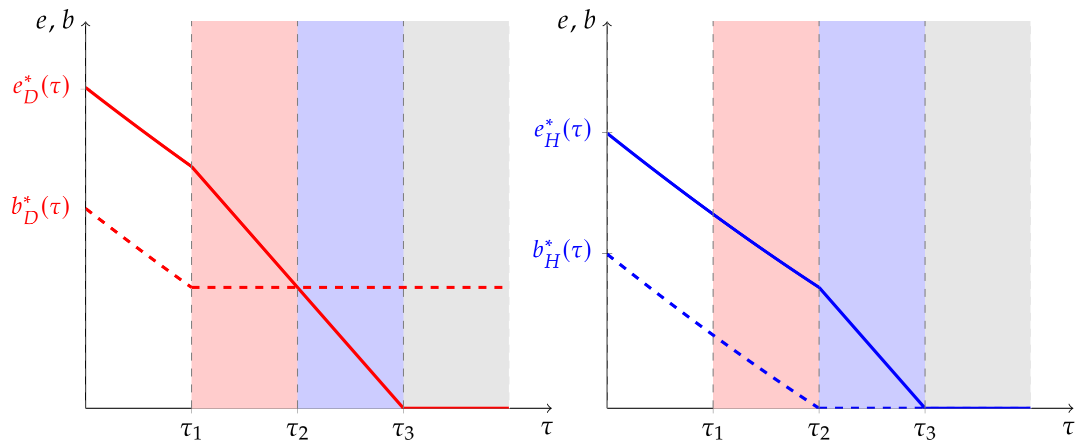
[19]The thresholds are fully characterized in the proof of Lemma 2 in appendix.

- *If $\tau \in [\tau_1, \tau_2)$, both types privately fund public services but only the honest type takes additional bribes to fund them.*

- *If $\tau \in [\tau_2, \tau_3)$, both types privately fund public services but neither type takes additional bribes to fund them.*

- *If $\tau \geq \tau_3$ neither type privately funds public services.*

Figure 1: Equilibrium funding and bribes



(a) Dishonest bureaucrat

(b) Honest bureaucrat

Notes. Funding and bribes of each type of bureaucrat as a function of tax. The solid lines display the funding ($e^*_\theta(\tau)$), while the dotted lines represent the bribes ($b^*_\theta(\tau)$). The left panel is for the dishonest type, while the right panel is for the honest type. In the red-shaded area, both types provide funding but only the honest type takes *additional* bribes. In the blue-shaded area, both types provide funding and neither takes *additional* bribes. In the gray-shaded area, neither type provides funding.

Figure 1 illustrates how the equilibrium funding and bribes identified in Lemma 2 interact with the politician's choice of tax.[20] When taxes are low, $\tau \leq \tau_1$, the bureaucrat funds a high level of public services so his budget constraint binds. The bureaucrats' funding sets the marginal benefit of funding (in terms of higher probability of retention) equal to the marginal cost (in terms of higher cost of taking bribes, since the constraint binds): $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e+\tau) = c(e - w_1, \theta)$ and $b^*_\theta(\tau) = e^*_\theta(\tau) - w_1$. Both the funding

[20]We focus on the case where the bureaucrat's funding alone cannot guarantee that the needs of the public will be met. That is, when $e^*_\theta(\tau) < \Psi$, $\forall \tau \in [0, +\infty)$, which occurs when $\phi\mu w_2 f(\Psi) - 1 < 0$. When $\phi\mu w_2 f(\Psi) - 1 \geq 0$, the bureaucrat's funding is potentially large enough to guarantee that the level of needs are met with certainty. In this case, the bureaucrat has no incentives to increase the level of public service beyond Ψ but the amount of funding still depends on the tax level in a similar way as in Lemma 2. The full characterization of the bureaucrat's private funding and bribes is provided in Lemma 5 in appendix.

level (solid red and blue lines in Figure 1) and the bribe level (dashed red and blue lines in Figure 1) are decreasing with taxes. When taxes satisfy $\tau \in (\tau_1, \tau_2]$, the bureaucrat funds a lower level of public services so only the budget constraint of the honest type binds. The honest type's funding and bribes solve the same conditions as when $\tau \leq \tau_1$, but the dishonest type does not take additional bribes, so $b^*_D(\tau) = c^{-1}(1, D)$. The red dotted line in the left panel of Figure 1 becomes independent of tax. The dishonest type's marginal cost of private funding is now only equal to the direct cost, 1, so $e^*_D(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$.[21] When $\tau \in (\tau_2, \tau_3]$, neither types' budget constraint binds. The honest type now stops taking bribes altogether, so the blue dotted line in the right panel of Figure 1 becomes 0, and her marginal cost of funding is now equal to 1. As a result, $b^*_H(\tau) = 0$, $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ for $\theta \in \{H, D\}$, and $b^*_D(\tau) = c^{-1}(1, D)$. Finally, when taxes are high, $\tau \geq \tau_3$, $e^*_\theta(\tau) = 0$ (both red and blue solid lines in Figure 1 are at 0), $b^*_\theta(D) = c^{-1}(1, D)$, and $b^*_\theta(H) = 0$.

There are two interesting takeaways from this result. First, the bureaucrat's decisions are determined by the level of tax. The bureaucrat's private funding of public services decreases in the level of formal taxation and is only positive if formal taxation is low. When taxation is very low, the bureaucrat takes more bribes than he would otherwise in order to fund public services. In this case, the level of tax therefore also affects bribes. Second, the amount of bureaucrat funding depends negatively on the cost of taking bribes (when the budget constraint binds) and positively on the observability of public services (ϕ). An increase in the observability of public services (ϕ) increases the marginal benefit of redistributing: meeting the needs of the population (which signals high ability) is more likely to lead to retention by the politician if the politician can observe it. In Figure 1, an increase in ϕ would lead the solid lines to shift to the right.

5.3 Politician's first-period strategy

The politician chooses a tax level τ, to maximize the citizens' expected utility, given the bureaucrat's best-responses $(b^*_\theta(\tau), e^*_\theta(\tau))$ and given her retention rule (from Lemma 1):

$$\max_\tau V(\tau) = \mathbb{E}_{\omega,\theta}\Big[\lambda F(\omega(\tau + e^*_\theta(\tau))) - \tau - \eta b^*_\theta(\tau) + \phi F(\omega(\tau + e^*_\theta(\tau)))\tilde{V}(1) + (1 - \phi F(\omega(\tau + e^*_\theta(\tau))))\tilde{V}(0)\Big]$$

[21] Note that $f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$ is well-defined since, by continuity of $f(y)$, there exists y such that $f(y) = \frac{1}{\phi\mu w_2}$ whenever $\phi\mu w_2 f(\Psi) - 1 < 0$.

where $\tilde{V}(1) = \lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)$ and $\tilde{V}(0) = \mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$ are the continuation values when the politician does or does not re-select the bureaucrat, respectively.

To simplify exposition, we make several parametric assumptions that we maintain throughout this section. First, we focus on the case where the dishonest bureaucrat's budget constraint is never binding.

Assumption 1. *The dishonest type can always cover his desired level of personal funding without additional bribes, $\Psi < w_1 + c^{-1}(1, D)$, and would provide enough funding to guarantee that the needs are met if the signal were perfectly revealing: $\mu w_2 f(\Psi) > 1$.*

The first part of the assumption eliminates the interval $[0, \tau_1]$ in Lemma 2 and allows us to focus on cases where the different behavior of the honest and dishonest bureaucrats creates a trade-off for the politician. The second part ensures that the derivative of the bureaucrat's objective function when $\phi = 1$ and $\tau = 0$ is increasing for any $e \in [0, \Psi]$.

Second, we assume that, in the absence of personal funding from the bureaucrat, it is optimal for the politician to choose the highest possible level of tax, $\tau = \Psi$ (and thus guarantee that the public service is provided since $F(\Psi) = 1$), given the politician's prior belief about the bureaucrat's ability (μ).

Assumption 2. *In the absence of private funding ($e_\theta = 0$), the marginal benefit of increasing the tax level at $\tau = \Psi$ is positive: $\mu\lambda f(\Psi) - 1 > 0$.*

Given the best-responses from the two types of bureaucrats identified in Lemma 2, the politician faces the following trade-offs. By choosing a low level of taxes, $\tau \in [0, \tau_2)$, she forces dishonest bureaucrats to redistribute a large portion of the bribes they take. The low official funding means that the public's needs are unlikely to be met which incentivizes bureaucrats to privately contribute large amounts to avoid being perceived as low ability. However, these incentives also drive honest bureaucrats to privately fund so much that their budget constraint becomes binding. As a result, a low level of official funding encourages honest bureaucrats to start taking bribes. This can be seen in the red-shaded area in Figure 1. When τ falls below τ_2, the honest bureaucrat (right panel) starts taking bribes. These bribes increase proportionally to funding. Since bribes are more costly to the citizens than taxes, the politician would be better off raising these funds through taxes. However, in this region, the dishonest bureaucrat (left panel) is also forced to redistribute some of their bribes: the red solid line (funding) increases as taxes go down while the red dotted line (bribes) remains constant. This is beneficial for the politician as it redistributes the rents extracted by the bureaucrat to the citizens.

If the politician increases taxes to $\tau \in [\tau_2, \tau_3)$, she reduces the need for private funding and honest bureaucrats no longer need to take bribes to fund public services. However, the lower need for private funding also implies that dishonest bureaucrats keep a higher share of bribes for themselves (the red dotted line in the left panel of Figure 1 is now above the red solid line). Finally, if the politician increases taxes to $\tau \geq \tau_3$, neither type of bureaucrat personally funds public services. Dishonest bureaucrats keep all the bribes that they extract, but the politician no longer relies on the willingness of bureaucrats to fund public services. At this point, the politician simply sets taxes at the maximum level, $\tau = \Psi$, given assumption 2.

In equilibrium, three types of policies can arise:[22]

1. **A formal fiscal policy:** the bureaucrat does not contribute to public services: $e^* = 0$ and only the dishonest type of bureaucrat takes bribes: $b^*_H = 0$, $b^*_D > 0$.

2. **An informal fiscal policy with low corruption:** both types of bureaucrats contribute to public services: $e^*_\theta > 0$, only the dishonest type of bureaucrat takes bribes: $b^*_H = 0$, $b^*_D > 0$.

3. **An informal fiscal policy with high corruption:** both types of bureaucrats contribute to public services: $e^*_\theta > 0$ and both types of bureaucrat take bribes, $b^*_H > 0$, $b^*_D > 0$.

In a formal fiscal policy, taxes are the highest and such that the politician is certain that the required level of public services will be met: $\tau^* = \Psi$. In an informal policy with low corruption, the politician chooses a lower level of tax than in a formal policy, and this level decreases even further in an informal policy with high corruption.

Our main result is that the share of dishonest bureaucrats, ν, the ease of monitoring public service provision, ϕ, and the cost of corruption to voters, η, determine which of the three policies is optimal. We begin by showing that the share of dishonest bureaucrats (ν) relative to the cost of corruption to voters (η) determines the politician's choice between the two types of informal fiscal policies. The observability of public services (ϕ) then determines whether this informal policy is better than a formal one.

[22]We show in the proof of Lemma 2 that these three types of policies cover all the possible combinations of equilibrium behavior from the bureaucrat.

Lemma 3. *There exist thresholds $\bar{\nu} \in (0,1)$ and $\underline{\nu} \in (0,1]$ on the probability that a bureaucrat is dishonest such that the politician prefers an informal policy with high corruption to one with low corruption if $\nu > \bar{\nu}$ and an informal policy with low corruption to one with high corruption if $\nu \leq \underline{\nu}$. The thresholds $\bar{\nu}$ and $\underline{\nu}$ are increasing in η.*

An informal policy with low corruption corresponds to a choice of tax on the second segment of the politician's payoff function (on $[\tau_2, \tau_3]$), which is strictly decreasing in τ (solid blue lines in Figure 2 and Figure 3 below).[23] Instead, an informal policy with high corruption corresponds to a choice of tax on the first segment of the politician's payoff function (on $[0, \tau_2]$, solid red line in in Figure 2 and Figure 3 below). If this segment is increasing, then it is better to increase taxes up until the point where the politician is choosing an informal policy with low corruption (i.e., $\tau = \tau_2$), so an informal policy with low corruption is better. If the first segment is decreasing, it is better to decrease tax down to zero, so an informal policy with high corruption is better.

Whether the segment is increasing or decreasing depends on the share of dishonest bureaucrats (ν). If the share of dishonest bureaucrats is high ($\nu > \bar{\nu}$), the effect of tax on funding and bribes is more likely to be the one displayed on the left panel of Figure 1, where decreasing taxes encourages the dishonest bureaucrat to redistribute more bribes. In this case, the politician's objective function decreases in tax on the first segment and the optimal informal policy is one with high corruption and no taxes. This is illustrated in Figure 2 below where the solid red line is decreasing and maximized at $\tau = 0$. Instead, when the share of dishonest bureaucrats is low ($\nu \leq \underline{\nu}$), the effect of tax on funding and bribes is more likely to be the one displayed on the right panel of Figure 1, where decreasing taxes forces the honest bureaucrat to take bribes to fund services (which is more costly than taxes). In this case, the politician's expected payoff is increasing in τ for $\tau \in [0, \tau_2]$ and the best informal fiscal policy is one with low corruption. This illustrated in Figure 3 below where the solid red line is increasing and maximized at $\tau = \tau_2$. We next solve for the optimal fiscal system in each of these two cases.

[23] In this region, both types of bureaucrats privately fund an amount $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi \mu w_2}\right) - \tau$, per Lemma 2, so the total amount of funding, $e^*_\theta(\tau) + \tau = f^{-1}\left(\frac{1}{\phi \mu w_2}\right)$ is independent of τ. Since bribes are also independent of tax in this region, increasing tax imposes a direct cost without generating additional funding for public services or decreasing bribes.

High share of dishonest bureaucrats

When the share of dishonest bureaucrats is high ($\nu > \bar{\nu}$), the optimal informal policy is one with high corruption and no taxes. Whether this informal policy is better for the politician than a formal policy depends on the observability of public service delivery.

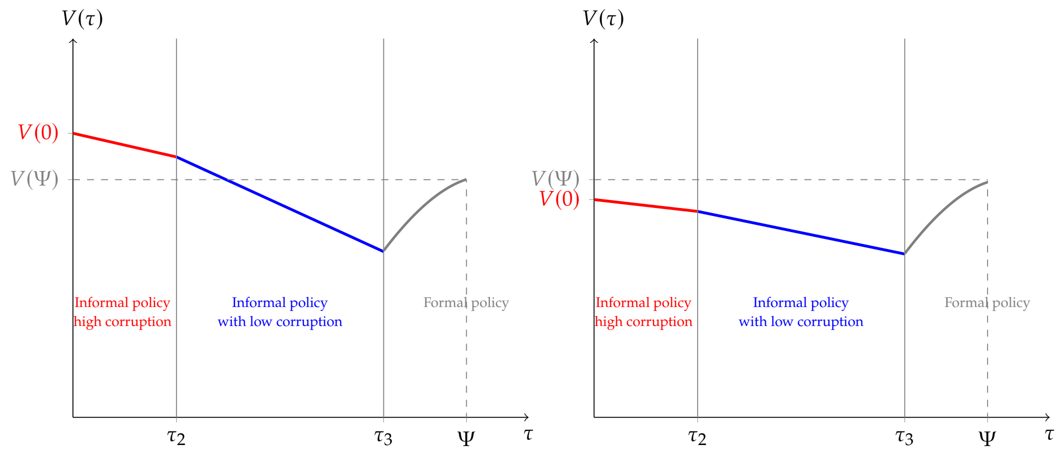
Proposition 1. *Suppose that $\nu > \bar{\nu}$, then there exists a threshold $\bar{\phi}_H \in [0, 1)$ on the observability of public services such that the politician chooses an informal policy with high corruption if $\phi > \bar{\phi}_H$. If the cost of corruption to voters is sufficiently low, $\eta < \bar{\eta}$, and the share of high-ability bureaucrats sufficiently high, $\mu > \bar{\mu}_H$, then this threshold is unique so the politician chooses an informal policy with high corruption if and only if $\phi > \bar{\phi}_H$, and a formal policy otherwise.*

Figure 2 shows how the politician's payoff changes as a function of tax, given the bureaucrat's strategic response. In both figures, the first vertical line corresponds to the level of tax above which the honest bureaucrat's budget constraint binds and the second vertical line corresponds to the level of tax above which bureaucrats do not want to fund any public services. The red and blue lines capture the politician's expected utility under an informal policy and the gray line captures her expected utility under a formal policy. The two figures illustrate the case where an informal policy with high corruption is optimal and the case where a formal policy is optimal. When ϕ is large (left panel), the bureaucrat's funding is higher, so the politician's payoff at $\tau = 0$ (where the bureaucrat's funding is highest, see Figure 1) is larger than her payoff at $\tau = \Psi$ (where funding comes entirely from official tax and therefore does not depend on ϕ). As a result, an informal policy is better. The reverse is true in the right panel where ϕ is smaller.

When the share of dishonest bureaucrats is high, the first segment (in red), is decreasing by Lemma 3. The second segment (in blue) corresponds to the case where neither type of bureaucrat's budget constraint is binding and is decreasing, as described above. The third segment, in gray, corresponds to the case where the bureaucrat does not redistribute funds ($\tau \geq \tau_3$). In this region, the politician's payoff is increasing in tax up to the point where she can guarantee to meet the public needs ($\tau = \Psi$) by assumption 2.

The optimal choice of policy can then be found by comparing the maximum payoff for the politician under an informal policy (the red line) with the maximum payoff under a formal policy (the gray line). When the observability of public service delivery is high ($\phi > \bar{\phi}_H$) the bureaucrat faces strong incentives to obtain bribes and redistribute them. When the share of corrupt bureaucrats ν is high relative to the cost of corruption η, this

Figure 2: High share of dishonest bureaucrats ($\nu > \bar{\nu}$)



(a) ϕ large (b) ϕ small

Notes. Objective function of the politician as a function of tax (τ) when $\nu > \bar{\nu}$. The left panel ($\phi \geq \phi_H$) shows the case where an informal policy is better, the right panel ($\phi < \phi_H$) shows the case where a formal policy is better.

redistribution outweighs the cost of encouraging honest bureaucrats to take additional bribes. As a result, the maximum of the politician's payoff under an informal policy ($V(0)$) is relatively high compared to a formal policy ($V(\Psi)$).

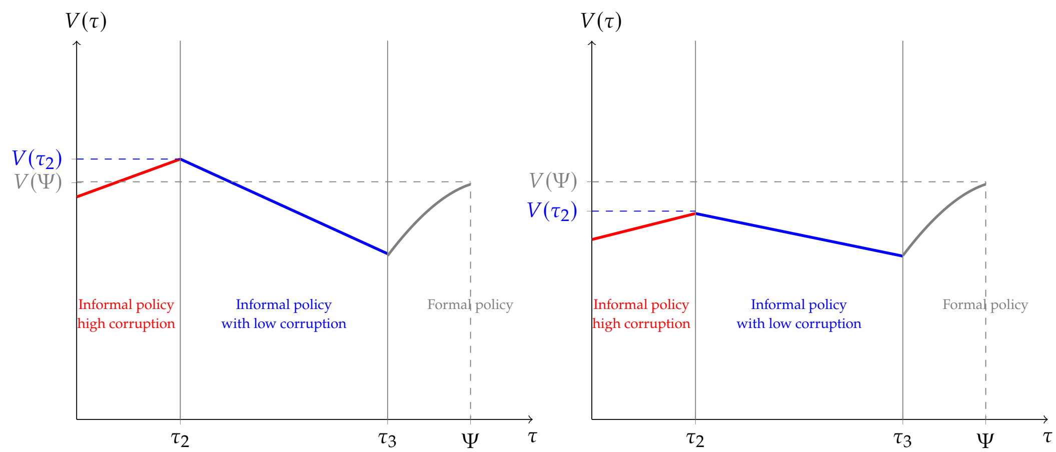
Low share of dishonest bureaucrats

When the share of dishonest bureaucrats is low, $\nu < \underline{\nu}$, the best informal policy is one with low corruption. Since there is no more corruption than in a formal fiscal policy, the choice between the two types of policies only depends on the observability of public services.

Proposition 2. *Suppose that $\nu < \underline{\nu}$, then there exists a threshold $\bar{\phi}_L$ on the observability of public services such that the politician chooses an informal policy with low corruption if $\phi > \bar{\phi}_L$. If the share of high-ability bureaucrats is sufficiently high, $\mu > \bar{\mu}_L$, this threshold is unique so the politician chooses an informal policy with low corruption if and only if $\phi > \bar{\phi}_L$, and a formal policy otherwise.*

Figure 3 illustrates the case where an informal policy with low corruption is optimal and the case where a formal policy is optimal when the share of dishonest bureaucrats is small. In both figures, the first vertical line corresponds to the level of tax above which the honest bureaucrat's budget constraint binds and the second vertical line corresponds to the level of tax above which bureaucrats do not want to fund any public services. The red and blue lines capture the politician's expected utility under an informal policy and the gray line captures her expected utility under a formal policy. In the left panel, ϕ is large so the bureaucrat's funding is relatively larger, even when her budget constraint is not binding. An informal policy (at $\tau = \tau_2$) is therefore better than a formal policy (at $\tau = \Psi$). In the right panel, ϕ is smaller so the reverse is true.

Figure 3: Low share of dishonest bureaucrats ($\nu < \underline{\nu}$)



(a) ϕ large (b) ϕ small

Notes. Objective function of the politician as a function of tax (τ) when $\nu < \underline{\nu}$. The left panel shows the case where an informal policy is better, the right panel shows the case where a formal policy is better.

When the politician chooses an informal policy with low corruption, honest bureaucrats fund public services without raising bribes. This happens when the tax levels are intermediate i.e. not too high (so that the bureaucrat wants to increase the level of public services) and not too low (as otherwise, the bureaucrat wants to fund such a large amount of public services that it is better to take bribes). This corresponds to another type of informal policy: one in which public services are funded through both personal donations and formal taxes but no bribes are extracted (except for the smaller share of dishonest bureaucrats). An example of such a policy is the case of school teachers or soldiers mentioned at the start of Section 2. For instance, school teachers in Mongolia who "use [their] own money for the school as the school fails to provide necessary materials for teaching" (Dashtseren, 2019) are less likely to raise money in the form of bribes than the police officers we study in India, yet also contribute financially to public service provision. Similarly, Ukrainian soldiers who “pay for their own uniforms, tools, cars, fuel, and spare parts"[24] are likely to fund these items from their own wage given limited bribe opportunities on the frontline.

Propositions 1 and 2 highlight how information frictions can sustain informal fiscal systems. Figure 4 illustrates these propositions. When corruption is widespread and the share of dishonest bureaucrats is high ($\nu \geq \bar{\nu}$) , the combination of adverse selection (the impossibility to identify dishonest bureaucrats) and moral hazard (the impossibility to control bribe taking) means that the politician cannot prevent corruption. When public service delivery is relatively easy to observe ($\phi \geq \phi_H$), it is therefore easier to incentivize bureaucrats to redistribute the bribes they are taking than from preventing them from taking bribes in the first place. The politician therefore prefers to fund public services through bribery than through taxes which leads her to choose an informal policy with high corruption (red area in Figure 4). Informal fiscal systems allow the politician to continue providing public services while avoiding a form of double taxation (bribes and formal taxes).

When the share of dishonest bureaucrats is low ($\nu < \underline{\nu}$) and public service delivery is easy to observe ($\phi \geq \phi_L$), the politician also wants to partially rely on bureaucrat funding, but does not want to encourage corruption. She therefore chooses an informal taxation with low corruption by setting taxes just sufficiently high to prevent the honest bureaucrat from taking bribes (blue area in Figure 4).

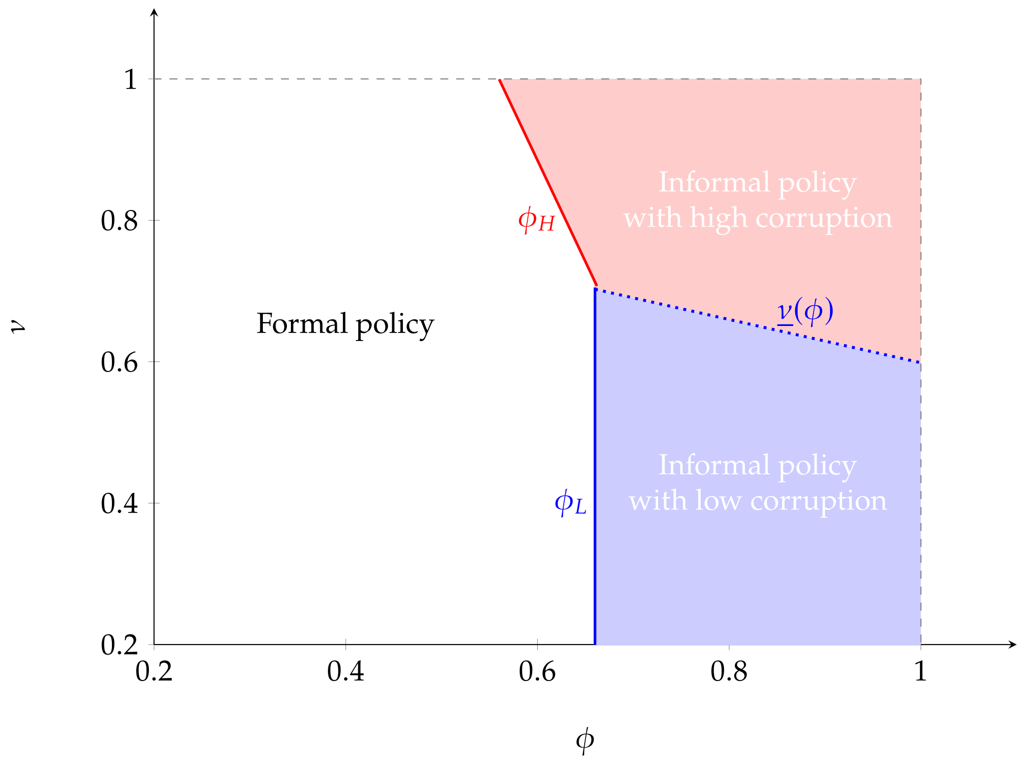
Finally, when public service delivery is harder to observe ($\phi < \min\{\phi_H, \phi_L\}$), the bureaucrats do not have sufficient incentives to deliver the right amount of public services, so the politician prefers a formal policy (white area in Figure 4).

5.4 Implications for bureaucratic selection and welfare

In this section, we explore the role informal fiscal systems can play in perpetuating corruption and their consequences for the welfare of citizens. We focus on the more interesting case of informal systems with high corruption throughout this section and therefore maintain that the share of dishonest bureaucrats is high enough ($\nu > \bar{\nu}$). We briefly discuss the the case of $\nu < \underline{\nu}$ in Section 5.6.

[24]Source: https://kyivindependent.com/ukrainian-soldiers-criticize-changes-to-combat-bonus-pay/, March 31, 2023.

Figure 4: Equilibrium policy given observability (ϕ) and dishonest bureaucrat share (ν)



Notes. In the white area, a formal policy is optimal since observability is low ($\phi < \min\{\phi_H, \phi_L\}$). In the blue area, the optimal policy is an informal one with low corruption since observability is high ($\phi \geq \phi_L$) but the share of dishonest bureaucrat is low ($\nu < \underline{\nu}$). In the red area, the optimal policy is an informal one with high corruption since observability is high ($\phi \geq \phi_H$) and the share of dishonest bureaucrat is high ($\nu \geq \bar{\nu}$). When $\nu \in (\underline{\nu}, \bar{\nu})$, Lemma 3 does not characterize the optimal informal policy as it depends on the shape of the function F. In this figure, an informal policy with high corruption is optimal in this region given the functional forms used ($F(x) = \frac{\ln(1+x)}{\ln(1+\Psi)}$).

5.4.1 Adverse selection in informal fiscal systems

In the previous section, we showed that, when the initial share of corrupt bureaucrats is high ($\nu > \bar{\nu}$), the politician prefers to implement an informal fiscal system with high corruption (provided that the observability of public services is high enough). We show that under such systems, dishonest bureaucrats are more likely to be retained in the next period than honest bureaucrats, even though the politician has no intrinsic preferences for corrupt bureaucrats and even though honesty and ability are independent.

Proposition 3. *Suppose that $\nu > \bar{\nu}$. If the observability of public services is sufficiently high ($\phi > \bar{\phi}_H$), a dishonest bureaucrat is more likely to be retained than an honest bureaucrat.*

When observability is high, the politician prefers an informal policy with high corruption by Proposition 1. Under such a policy, the dishonest bureaucrat chooses a higher level of personal funding than an honest bureaucrat ($e_D^* > e_H^*$). The marginal benefit of private funding is the same for both types, but the marginal cost of the honest bureaucrat is higher than that of the dishonest bureaucrat (since the honest bureaucrat has a higher marginal cost of taking bribes to fund services than the dishonest one). A higher level of funding increases the probability that the citizens' needs are met which serves as a signal of high ability to the politician. As a result, the politician is more likely to get a positive signal of the bureaucrat's ability when the bureaucrat is dishonest than honest and therefore more likely to retain dishonest bureaucrats.

While we limit the model to two periods and abstract from corruption opportunities in the second period, the main intuition would carry over to an infinitely repeated version of the game: a low level of tax would force dishonest bureaucrats to redistribute the bribes they take and encourage honest ones to take additional bribes. As shown in Lemma 3, a higher share of dishonest bureaucrats makes the politician more likely to choose an informal fiscal system when facing this trade-off. As a result, Proposition 3 implies that informal fiscal systems can be self-reinforcing: they arise when the share of dishonest bureaucrats is high and they are more likely to lead to the retention of dishonest bureaucrats.

5.4.2 Welfare implications

In our model, the politician faces some agency frictions due to moral hazard and adverse selection which allow the bureaucrat to take bribes. Informal fiscal systems offer a second-best alternative to attenuate the effect of these agency frictions but can also introduce additional distortions because the funding of public services is delegated to bureaucrats.

To understand the consequences of these frictions, we begin by analysing the first-best: a politician who faces no moral hazard (so she can choose any b and e subject to the constraint that $e \leq w_1 + b$), and faces no adverse selection (so she can perfectly select high-ability bureaucrats). In the first-best outcome, the politician funds the public good through formal taxes and donations from the bureaucrat but not bribes. Since $\eta > 1$, funding the good through taxes is less costly than funding it through bribes. The politician makes the bureaucrat redistribute his wage and sets $e_{FB} = w_1$ (since this comes at no cost to the utility of the voters), but not provide any additional funding, so $b_{FB} = 0$. Instead, she sets taxes at $\tau_{FB} = \Psi - w_1$. The expected amount of public services in the first-best is $y_{FB} = \mu\Psi$ and comes at a cost $\mu(\Psi - w_1)$ to citizens.[25]

Comparing these outcomes to the case where the politician cannot impose the choice of b or e on the bureaucrat and cannot observe the bureaucrat's type, as in Proposition 1, reveals the welfare impact of informal fiscal systems:

Proposition 4. *Agency distortions can make informal fiscal systems socially desirable. However, corruption is higher and the amount of public services is weakly lower in informal fiscal systems than in the first best. When the the amount of public services is the same as in the first best, the cost of funding public services is higher in informal fiscal systems than in the first best.*

Agency frictions have both a direct impact on welfare, by increasing corruption and the cost of funding public services, and an indirect impact on welfare, by changing the policy chosen by the politician. If the politician chooses a formal policy, the amount of public services remains the same as in the first best, $y_{\text{Formal}} = \mu\Psi$, but two distortions arise. First, dishonest bureaucrats take bribes, so corruption increases relative to the first best to $b_{\text{Formal}} = c^{-1}(1, D)$. Second, the politician has to raise taxes without knowing the bureaucrat's ability and cannot force the bureaucrat to redistribute his wage so the expected cost of funding increases to $\Psi > \mu(\Psi - w_1)$. As a result of these distortions, the politician might prefer to implement an informal fiscal policy (Proposition 1). When she chooses an informal policy, the amount of public services drops to $y_{\text{Informal}} = \mu\left(\nu e_D^*(0) + (1-\nu)e_H^*(0)\right)$ and corruption increases to $b_{\text{Informal}} = \nu c^{-1}(1, D) + (1-\nu)b_H^*(0)$ relative to the first best. However, given the agency frictions, this maximizes the utility of the voters by forcing the bureaucrats to redistribute some bribes and thus avoiding a form of double taxation.

5.5 Political distortions and incidence

To understand how political distortions can lead to these systems, we extend the model and introduce two groups of citizens: the rich, R, and the poor, P. The two groups differ in how much income they have, with the rich earning higher income $W_R > W_P$, and in how much they value the public good, with the rich valuing it less $\lambda_R < \lambda_P$ (for instance because they can access some of these services privately). Finally, we modify the model to allow the politician to choose a proportional income tax, rather than a lump-sum tax: the politician chooses $t \in [0, 1]$ and each group $i \in \{R, P\}$ pays $t \times W_i$ in tax so that the total

[25]See Lemma 7 in appendix for details.

amount of tax raised is $t \times (W_R + W_P)$. Since we take income as exogenous, a proportional tax does not introduce any distortion and is therefore equivalent to a lump-sum tax.[26]

We assume that the groups are of equal size and do not differ in any other way. In particular, we assume that they both bear an equal share of the bribes obtained by the bureaucrat: the cost of a level b of bribes to each group is $\frac{\eta b}{2}$. We maintain the assumption that using bribes to fund public services is more distortionary than taxes in aggregate: $\eta > 1$. Finally, we also continue to assume that, in the absence of private funding, it is optimal for a politician to provide sufficient funds to guarantee that the public service will be delivered. This is ensured with an assumption equivalent to assumption 2:

Assumption 3. *In the absence of private funding, the marginal gain to group R of increasing tax is positive for all* $t \in \left[0, \frac{\Psi}{W_R + W_P}\right]$, $\mu \lambda_R f(\Psi) - \frac{W_R}{W_R + W_P} > 0$.[27] *Moreover, the voters can afford to fund* Ψ *in aggregate:* $\Psi < W_R + W_P$.

Throughout this section, we consider a politician who favors group R. This could be the results of a higher turnout among the rich or the fact that the rich can exert more influence on politicians through other means such as campaign contributions. We show that these political distortions can lead the politician to choose an informal fiscal system, even in situations where formal fiscal systems are socially optimal. To analyze these distortions, we compare the policy chosen by a social planner who maximizes the sum of the two groups' utilities with the equilibrium choice of the politician favoring group R.[28] While an informal fiscal policy can be chosen in both cases, the range of parameters for which they are chosen differs. We define the thresholds ν_{SP}, ν_R, η_{SP}, η_R, μ_{SP}, and μ_R as the equivalents of $\bar{\nu}$, $\bar{\eta}$, and $\bar{\mu}_H$ in Lemma 3 and Proposition 1.

Proposition 5. *Consider a politician and a social planner who both face moral hazard and adverse selection. Suppose that* $\nu > \max\{\nu_R, \nu_{SP}\}$, $\eta < \min\{\eta_{SP}, \eta_R\}$, *and* $\mu > \max\{\mu_R, \mu_{SP}\}$. *The range of the observability parameter,* ϕ, *for which an informal fiscal system is chosen is larger for a politician favoring group R than for a social planner who treats both groups equally.*

Political pressure can lead the politician to finance public goods through bribery rather than taxes even when it is not socially optimal because group R bears a higher share of the

[26]We abstract from the usual distortions on labour supply or consumption that taxes induce to focus on the existence of informal system. Distortions that make formal taxes less desirable would make informal systems relatively more desirable.

[27]Note that, since $\lambda_P > \lambda_R$ and $W_P < W_R$, assuming that this inequality holds for group R implies that it also holds for group P.

[28]The equilibria in these two cases are characterized in Lemmas 9 and 10 in the appendix.

formal tax burden under a proportional tax while valuing the public services relatively less. This makes the informal policy relatively more attractive to that group. As a result, the public service provision decreases relative to the social planner's choice and the source of funding (bribes) is socially inefficient.

Proposition 4 and 5 imply that both information frictions (moral hazard and adverse selection) and political frictions (favoring one group of voters) can make informal fiscal systems more likely. This highlights an important interaction between political and agency frictions in the presence of informal fiscal systems. The existence of agency frictions makes informal fiscal systems desirable (both for the politician and the social planner): it can be optimal to incentivize dishonest bureaucrats to redistribute the bribes they take if they cannot be prevented from taking bribes in the first place. But the presence of political frictions exacerbates these incentives: a politician might prefer an informal fiscal system even when the observability of public services is too low for informal systems to be socially optimal ($\phi < \bar{\phi}_{SP}$). Since group R values the public service less than group P ($\lambda_R < \lambda_P$) but bears a relatively higher share of the tax burden ($W_R > W_P$), informal fiscal system with a more equal distribution of bribes and lower public services are favored by group R voters. In turn, informal fiscal systems create further agency distortions. Beside the increase in adverse selection that these systems introduce, as discussed in Section 5.4.1, the provision of public services is delegated to bureaucrats, so the public service can be under provided and corruption increases.

Incidence. Informal fiscal systems will generally have a different incidence than formal fiscal systems. In a formal system, the proportion of public services that is funded by different groups simply corresponds to the amount of tax each group pays relative to the total amount of taxes: $\mathcal{I}_i^{\text{Formal}} = \frac{t^* W_i}{t^*(W_R+W_P)} = \frac{W_i}{W_R+W_P}$, $\forall i \in \{R, P\}$. Each group therefore bears a burden of tax proportional to their income. Instead, when the politician chooses an informal policy, the proportion of public services funded by a group depends on the amount funded by bribes. Since the tax rate is zero in the optimal informal system, the incidence becomes $\mathcal{I}_i^{\text{Informal}} = \frac{t^* W_i + \frac{\eta}{2} e^*}{t^*(W_R+W_P)+\eta e^*} = \frac{1}{2}$, $\forall i \in \{R, P\}$. When the rich are more politically-influential and the observability of public services, ϕ, is large enough, the politician chooses an informal system. This system leads the poor to bear a relatively higher fiscal burden ($\frac{1}{2} > \frac{W_P}{W_R+W_P}$) compared to a formal system and the rich to bear a lower fiscal burden ($\frac{1}{2} < \frac{W_R}{W_R+W_P}$). In this case, informal fiscal system are therefore regressive relative to formal fiscal systems.[29]

5.6 Discussion

The baseline model provides a framework for understanding when informal fiscal systems emerge and how they operate. In this section, we explore several extensions explaining how our model connects with related practices, such as user fees and informal taxation, or exploring the robustness of our results to certain assumptions.

User fees as informal fiscal systems. User fees differ from informal fiscal systems in three fundamental ways. First, explicit user fees are generally seen as legitimate, which implies that bureaucrats would not typically be punished for asking citizens to pay them, unlike bribes. Second, bureaucrats can condition access to the public service on the payment of a user fee. This implies that, even implicit (illegal) user fees, can be more easily extracted from citizens than bribes (as the bureaucrat has direct bargaining power) but also that a citizen's willingness to pay for a service puts an upper bound on the amount of rents that the bureaucrat can extract from user fees. In an informal fiscal system, this might not be the case if the bureaucrat extracts bribes from unrelated services. Finally, since the bribe can be extracted from unrelated services, informal fiscal systems inherently allow for some redistribution, which is not possible with user fees.

The first two aspects directly affect the baseline model. To understand their implications, we consider an alternative model in which bureaucrats cannot use bribes to fund public services, but can instead charge a user fee. Charging the user fee comes at no cost to the bureaucrat, unlike bribes, to reflect both the facts that they are seen as more legitimate and that the bureaucrat has more bargaining power.[30] However, like the bribes, user fees relax the bureaucrat's budget constraint for providing personal funding. The marginal cost of the user fee to the citizen is the same as that of taxes, but, unlike taxes, user fees are

[29]More generally, if each group bears a cost η_i of corruption, the incidence of an informal fiscal system on group i is $\frac{\eta_i}{\eta_R+\eta_P}$. It is then also possible for informal systems to be more progressive than formal systems. If the poor are more politically-influential and the rich pay a larger proportion of bribes than the poor, then the fiscal burden can fall disproportionately on the rich relative to a formal system. Finally, if informal systems act as de facto user fees they can have a more neutral incidence. For instance, if only petrol station owners benefit from additional police patrols, providing free petrol is a way to privately fund the provision of policing. In this case, the incidence of funding falls on the group who accesses the service, which is also the only group that benefits from it, so informal system have no effect on redistribution.

[30]The assumption that user fees come at no extra costs is more applicable to explicit user fees than implicit ones, but the intuition would continue to hold if the cost is just lower than that of bribes.

only reflected in the public service indirectly through the bureaucrat's choice of personal funding.

We solve this model in Appendix Section A.2 and show that this model of user fees has different implications than our model of informal fiscal systems. First, the fact that user fees are easier to extract and more legitimate means that the cost of extracting them depends less on the bureaucrat's honesty. As a result, the politician does not necessarily face a trade-off between forcing dishonest bureaucrats to redistribute bribes and encouraging honest bureaucrats to take bribes, so the emergence of user fees does not necessarily depend on the share of dishonest individuals in the bureaucracy (Lemma 3). Second, the decision to re-select bureaucrats in the second period depends less on their honesty, so the self-fulfilling aspect of informal fiscal systems (Proposition 3) is less important with user fees. Finally, the amount of user fees that bureaucrats can extract, and therefore the rents that they keep, is bounded by the value that citizens assign to the service.

The third distinction, that user fees do not allow for redistribution, is the most important conceptual difference. To illustrate it, we apply our model of user fees to the case of multiple groups presented in Section 5.5. Suppose that the public service's benefits are so low for group R that group R is not willing to pay a user fee to access it but group P is willing to pay a user fee (i.e., $\lambda_R = 0$ but $\lambda_P > 0$).[31]. In that case, only group P benefits from the service, and only that group indirectly funds the service through user fees, so there is no redistribution (other than through formal taxes, which can be zero in equilibrium). With informal systems instead, the bureaucrat could extract bribes from an unrelated service which is valuable to group R and use these bribes to fund the service which is valuable to group P, thus allowing redistribution.

In fact, this corresponds to several of the examples we document, which cannot be described as instances of user fees. For instance, when bureaucrats fund food banks in the Pakistan case, the users of the food bank receive the food for free and are therefore not paying for the public service. Instead, these bureaucrats frequently obtain bribes from delivering land titles, so the food banks are indirectly funded by wealthier land owners.

Relationship with informal taxation. Informal taxation and informal fiscal systems are not mutually exclusive. In fact, our data from Pakistan shows that several of the services are funded by both bureaucrats and local philanthropists (see Table 2 and Table 3).

[31]For instance, if the service is public healthcare, the rich might not be willing to pay for it if they can afford better service from private providers.

To understand the relationship between informal taxation and bureaucrat funding, we extend our model in Appendix A.3 to allow citizens to contribute to the public service. We assume that citizens can choose how much to contribute after observing whether the government and the bureaucrat funding meet the minimum threshold $\bar{y}$. This captures the fact that informal taxation can take advantage of local information available to the local population but not to the government or the bureaucracy. On the other hand, we assume that the citizen funding's marginal cost is larger than that of taxes ($\rho > 1$). This captures the fact that funding by citizens might not benefit from the economies of scale of the government or the know-how of bureaucrats. The politician chooses the level of formal taxes, τ, anticipating both the bureaucrat's funding and the citizen's funding.

We show that, in this model, informal taxation can co-exist with bureaucrat funding. The bureaucrat still funds services in an attempt to signal his ability to the politician, while citizens "top up" the bureaucrat's funding to ensure the right level of public services are provided. We show that, the lower the bureaucrat's incentives are, the higher the level of informal taxation. This suggests that informal taxation can complement bureaucrat funding, especially in contexts where local information is important or the bureaucrat's incentives are not strong enough. If the cost of citizen funding, ρ, is too high, the politician raises formal taxes to guarantee the service provision, without requiring informal funding from either bureaucrats or citizens.

Heterogeneity in ability vs. heterogeneity in honesty. In the model, bureaucrats differ both according to their honesty and their ability. Since bureaucrats know their honesty but not their ability, the differences in the behavior of different types of bureaucrats depends on their honesty. A natural question is therefore whether both dimensions of heterogeneity are necessary.

In Appendix Section A.4, we show that heterogeneity in ability, in addition to honesty, is essential for the model's results, and that without this heterogeneity, informal fiscal systems would be unlikely to arise. Indeed, there are several equilibria in which both types of bureaucrats provide no funding at all. Moreover, the equilibria in which bureaucrats do provide funding become sensitive to the assumption that the politician pays no cost for replacing bureaucrats and has no intrinsic preferences for different bureaucrats. Intuitively, without heterogeneity in ability, the politician's re-selection decision becomes independent of the bureaucrat's performance, since performance does not provide any relevant information about the politician's expected second-period payoff. When re-selection is independent of performance, bureaucrats have no incentives to personally fund public services to increase the likelihood that the citizens' needs are met.

More generally, the heterogeneity in ability can create a trade-off for the politician who might prefer to re-select a high-ability bureaucrat even if they are more likely to be dishonest, as long as this bureaucrat can deliver much needed public services. While this trade-off is muted in the model, as we assume that there is no corruption in the second period, we show in Appendix A.4 that when the politician prefers to re-select an honest bureaucrat (e.g., because bribe-taking is possible in the second period) but there is no heterogeneity in ability, the bureaucrats would prefer not to fund any public services so the only possible policy would be a formal policy. This suggests that informal policies are likely to arise in contexts where there is a strong need for competent bureaucrats to deliver public services and this need outweighs the potential costs of re-selecting dishonest bureaucrats.

Patronage and collusion. A common feature of the contexts we study is the presence of informal connections between bureaucrats and politicians. Two aspects of these informal connections are relevant to our study. The first is collusion: politicians, or senior civil servants, often expect to receive a portion of the bribes collected by bureaucrats (Brierley, 2020; Sanchez De La Sierra et al., 2024). The second is patronage: politicians might offer valuable positions in the bureaucracy to influential citizens, or promote bureaucrats that they are connected with, in exchange for political support (Colonnelli et al., 2020; Callen et al., 2023; Toral, 2024). We consider two extensions of the model which incorporate collusion and patronage, respectively.

In appendix A.5, we extend our model to force the bureaucrat to transfer a portion of his bribes to the politician. This has two conflicting effects: on the one hand, it reduces the marginal benefit of taking bribes since the bureaucrat does not get to keep all the bribes. This makes the bureaucrat's budget constraint more binding, reduces their public service funding, and, as a result, makes informal systems less appealing to politicians. On the other hand, since the politicians receives a portion of the bribes, she would like to encourage bribe taking. This makes informal systems more appealing to the politician. The presence of collusion therefore has an ambiguous effect on the existence of informal fiscal systems.

In appendix A.6, we consider an extension in which the politician is connected to some of the bureaucrats. When the politician re-selects a connected bureaucrat, she receives some political benefit from retaining the bureaucrat independent of the bureaucrat's performance. When the politician is connected to the bureaucrat in office, patronage leads her to re-select the bureaucrat even after poor performance. This reduces the bureaucrat's incentives to fund public services, and make informal fiscal systems less appealing. However, when the politician is not connected to the bureaucrat in office, but connected to a bureaucrat in the replacement pool, the performance threshold for being re-selected can increase which can lead the bureaucrat to increase her funding and make informal systems more desirable.

Naive voters, responsibility for corruption, and lower cost of corruption to voters. In the model, we assume that the marginal cost of corruption to citizens is greater than the marginal cost of taxation: $\eta > 1$. This assumption captures the idea that corruption can be more distortionary than taxes but also requires that the politician is held accountable for all the corruption. In practice, the politician might be able to take credit for the public services provided by bureaucrat, but avoid getting blamed by citizens for the bribes taken by bureaucrats. Alternatively, corruption could be perceived by voters as being less costly than taxes if they trust local bureaucrats more than the central government. Both cases can be captured by assuming that $\eta < 1$ in the politician's objective function. In appendix A.7, we show that, when $\eta < 1$, the politician's incentives to implement an informal fiscal system with high corruption increase. This change in incentives takes place through two mechanisms. First, the politician is more likely to choose an informal policy with high corruption than one with low corruption (i.e., the threshold on the share of dishonest bureaucrats above which the politician prefers high corruption ($\bar{\nu}$) decreases). Second, the value of an informal policy with high corruption increases and therefore is more likely to be above that of a formal policy (i.e., the threshold on the observability of public services above which the politician prefers an informal policy ($\bar{\phi}_H$) decreases).

Altruism and intrinsic motivation. Besides career concerns, another motivation for bureaucrats could be altruism or intrinsic motivation. If altruism is uncorrelated with honesty, one could simply re-interpret the function $\phi F(e + \tau)$ as capturing the intrinsic motivation of the bureaucrat. Higher intrinsic motivation would make informal systems more likely to be chosen over formal systems by the same logic as Propositions 1 and 2. However, informal systems could now lead to the positive selection of intrinsically motivated bureaucrats who provide more personal funding (but also still lead to the adverse selection of dishonest bureaucrats since their cost of funding services through bribes is lower). If intrinsic motivation is positively correlated with honesty, there could be a separating equilibrium in which honest and intrinsically motivated bureaucrats personally fund services and take no bribes, while dishonest bureaucrats with low motivation do not fund services and take bribes. While these alternative motivations are plausible in some contexts, they do not align well with responses to our surveys in Pakistan, where none of the supervisors (and only 30% of the bureaucrats) reported concerns for the local population as a reason for providing personal funding (Table 4).

Lower optimal tax rate. Assumption 2 implies that it is optimal to set the tax at the maximum level, Ψ, in a formal system and in the second period. One implication is that the optimal tax in a formal system is independent of the observability of public services which simplifies the proof of Propositions 1 and 2. Relaxing this assumption could mean that the threshold on ϕ for an informal policy to be preferred may not be unique. However, it would still be the case that an informal policy is preferred for a sufficiently high level of observability, ϕ. As ϕ increases, the private funding provided by bureaucrats ultimately gets very close to the optimal formal tax while the cost of funding remains below it since some funding comes from the dishonest bureaucrat's existing bribes.

Welfare implications with low share of dishonest bureaucrats. Throughout Section 5.4.2, we focused on the case where the share of dishonest bureaucrats is high. When the share of dishonest bureaucrats is low, the politician prefers an informal policy with low corruption over one with high corruption. In this case, informal systems do not lead to more adverse selection (Proposition 3) as both types provide the same amount of funding.[32] The second part of Proposition 4 continues to hold, since public services can be under provided when delegated to the bureaucrat, but the first part does not, since the first-best also involves some redistribution from the bureaucrat which is not driven by agency distortions. Proposition 5 also continues to hold as group R still benefits relatively less from a formal fiscal system.

[32] However, this is partly driven by the assumption that there are no opportunities for corruption in the second period. If there were, the dishonest bureaucrat would have more incentives to provide funding and therefore be more likely to be retained.

6 Conclusion

Developing countries worldwide face substantial hurdles in their attempts to provide public goods. We describe a method through which some governments handle these constraints: through an informal fiscal system in which local bureaucrats are expected to finance public services out of their own pockets. We document the existence of such systems in a large bureaucracy in Pakistan, showing that bureaucrats most likely make up for these shortfalls in official funds through rent extraction.

Our model describes the conditions under which governments might prefer to implement low formal taxes and encourage bureaucrats to fund public services. We show that these systems are more likely to arise when corruption is widespread but the public services that bureaucrats can fund are relatively easy to observe, and when politically powerful groups bear a relatively larger share of the formal tax burden than of the cost of corruption.

The existence of informal fiscal systems can explain the joint persistence of corruption and low fiscal capacity. Because governments can rely on corruption to fund public services, they have limited incentives to punish it and to invest in fiscal capacity. The costs of such systems can be large, as (somewhat) legitimized rent extraction and low monitoring may lead to high levels of corruption, even if some funds are returned in the form of public services. Moreover, distributional consequences are unavoidable if only some parts of the population are targeted for rent extraction and the ability of governments to redistribute across space is restricted with necessarily local informal fiscal systems. How and when such discretionary, informal systems transition to programmatic formal systems are questions for future research.

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Table 1: Examples of informal fiscal systems across the developing world

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Bangladesh	U4 Expert Answer	https://www.u4.no/publications/overview-of-corruption-within-the-justice-sector-and-law-enforcement-agencies-in-bangladesh.pdf	2012	"Due to the limited amount of funds allocated, the courts suffer from lack of basic necessities, such as stationery and other office supplies. It has been reported that these shortfalls are often met by bench assistants and office staff. To cover these expenses, court officers can condone or overlook demands for money from the litigants by lower level court staff."	1	1
Liberia	Liberian Observer	https://www.liberianobserver.com/liberia-no-operational-funds-police-breeds-corruption	2022	"although police corruption well exists, police officers "ask families of victims and survivors of rape for gas money not because they wish to enrich themselves, but because their government fails to provide the basic operational funding needed to fill a tank of gas to respond to emergencies.""	1	1

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Mozambique	Danish Institute for International Studies	https://www.diis.dk/en/research/the-predicament-of-mozambiques-police-force	2023	"While many police officers often resort to side businesses alongside their official duties to make ends meet, some have gained notoriety for transgressing the law, either by extorting money from citizens or requesting 'refrescos' (soft drinks) – a euphemism often used when police seek small sums of money from people during patrols and various tasks...“When we arrived at the police station, the police simply stated that they would not initiate any legal proceedings. Instead, they told us, ”You just need to pay us 500 meticais, and then you can go home. We need the money because we haven't received our salaries since May.”"	1	1

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Nigeria	Vanguard	https://www.vanguardngr.com/2021/04/how-police-funding-challenge-can-be-tackled-lawyers/	2021	"the rank and file depend on extortion from members of the public to buy uniforms and other supplies."	1	1
Yemen	Yemen Policy Center	https://www.yemenpolicy.org/policing-in-a-fragmented-state-resilience-of-local-state-institutions-in-taiz/	2022	"According to a 2019 Yemen Polling nationwide survey, 78 percent of Taiz residents believe the police would be less corrupt if they were paid more. Against this backdrop, police stations have developed 'new' services in an effort to mobilize revenue."	1	1
Tajikistan	U4 Expert Answer	https://www.u4.no/publications/overview-of-corruption-and-anti-corruption-in-tajikistan.pdf	2013	"Law enforcement agencies lack adequate resources and police salaries are low which creates incentives to demand bribes."	1	1

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Zimbabwe	News Day	https://www.newsday.co.zw/local-news/article/19306/underfunding-fuelling-police-corruption	2021	"police officers have become corrupt because their parent ministry is underfunded to meet their needs."	1	1
Ecuador	New York Times	https://www.nytimes.com/2023/07/12/world/americas/ecuador-drug-cartels.html	2023	"A lack of funds, the officer explained, meant officers paid out of their own pockets to fix their vehicles. Instead of radios, they used their own phones to communicate."	1	Unknown
Turkmenistan	Global Security	https://www.globalsecurity.org/military/library/news/2023/02/mil-230223-rferl03.htm	2023	"Being a state worker in Turkmenistan comes with many strings attached, with officials often ordering employees to pay for various government-backed charities and projects, to walk in parades, or help clean the streets... State workers in the provinces must pay for the refurbishment of their office buildings"	1	Unknown

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Russia	The Moscow Times	https://www.themoscowtimes.com/2022/05/20/we-have-to-buy-everything-ourselves-how-russian-soldiers-go-off-to-fight-a77751	2022	"If they issue you a field uniform, you're in luck — you can save some money. We still have to buy the jacket and pants, at least as a change of clothing... I'll be happy if our outlay on the uniforms pays off and we don't get screwed out of our paycheck," said one contract serviceman with Russia's National Guard (Rosgvardia)."	1	Unknown
Ukraine	Kyiv Independent	https://kyivindependent.com/ukrainian-soldiers-criticize-changes-to-combat-bonus-pay/	2023	"Yet service members said .... they often can't get the gear they need on time or at all. As a result, many pay for their own uniforms, tools, cars, fuel, and spare parts.."	1	Unknown
Syrian Arab Republic	Relief Web	https://reliefweb.int/report/syrian-arab-republic/afraid-go-class-ten-years-start-syria-crisis-children-and-teachers	2021	"We can't provide school supplies like stationery, bags and notebooks. Teachers have been working voluntarily without pay for several years."	1	Unknown

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Mexico	Aljazeera	https://www.aljazeera.com/news/2018/8/2/mexico-police-officers-underpaid-under-equipped	2018	"It's not just weapons and munitions that are lacking...'My superiors always told me the same thing – put up with it or buy it [a new battery] yourself,' he told Al Jazeera. 'Just like everything else,' he said. 'If a tyre went flat, you had to pay for the patch; change of oil – we did it ourselves; when there wasn't enough gas, we needed to buy it ourselves........A quarter of the almost 5,000 state and federal officers Causa en Comun questioned said they had to pay for car repairs from their own pocket, 41 percent said they had to buy boots from their own salary and 38 percent had to pay for their own uniforms."	1	2
Mongolia	Asian Journal of Management Sciences & Education	http://www.ajmse.leena-luna.co.jp/AJMSEPDFs/Vol.8(4)/AJMSE2019(8.4-05).pdf	2019	"Teachers use own money for the school as the school fails to provide necessary materials for teaching, mainly due to the limited financial sources for operational costs."	1	2

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Costa Rica	StateUni-versity.com Education Encyclopedia	https://education.stateuniversity.com/pages/303/Costa-Rica-TEACHING-PROFESSION.html	2023	"Teacher salaries account for more than 50 percent of the education budget, but the salaries are low when compared to those of other public employees. Additionally, many teachers must buy supplies and pay for school repairs out of their own salaries."	1	2
Philippines	Asian News Network	https://asianews.network/philippines-teachers-taking-out-loans-to-prepare-classrooms/	2022	"Teachers Dignity Coalition chair Benjo Basas on Tuesday cited reports of teachers having to take out loans in order to buy paint, iron sheets and glass panes to get their class-rooms ready."	1	2
Papua New Guinea	The National	https://www.thenational.com.pg/address-teachers-concerns/	2023	"They [teachers] use their salaries to buy school supplies and/or provide logistics when TFF is not yet available (in rural schools)"	1	2

Table 1 – *Continued from previous page*

Country	Newspaper	Web Link	Year	Relevant cite	Are bureaucrats expected to provide public services in the absence of official funds? (No=0, Yes=1)	Source of funds (Bribes=1, Wages=2, Unknown)
Vietnam	Vietnam Net	https://vietnamnet.vn/en/teachers-pay-for-class-materials-from-salaries-E1809.html	2010	"Teachers pay for class materials from salaries ...it was not unreasonable to expect teachers to make their own props and that every term kindergartens awarded prizes to the most creative teachers. But she said teachers should not be expected to have to spend their own money on materials."	1	2

Table 2: Provision of public goods and services by local bureaucrats without official funds

	Mean	N	SD
	(1)	(2)	(3)
Panel A: Bureaucrat perspective			
Whether local bureaucrats provide underfunded public services (proportion who agree)	0.82	750	0.39
Proportion of respondents who reported a positive amount of funds supplied by:			
Local bureaucrats	1.00	618	0.05
Government funds	0.02	618	0.15
Local philanthropists	0.30	618	0.46
NGO	0.21	618	0.41
Other	0.00	617	0.00
Share of local bureaucrat's total expenditure			
Expenditure on unofficial public services	15.45	557	21.77
HH consumption	46.21	556	16.79
Children expenditure	27.44	557	11.49
Travelling	13.60	557	6.60
Other	2.86	703	5.65
Panel B: Supervisor perspective			
Whether local bureaucrats provide underfunded public services (proportion who agree)	0.98	35	0.14
Proportion of respondents who reported a positive amount of funds supplied by:			
Local bureaucrats	0.89	33	0.31
Government funds	0.78	33	0.42
Local philanthropists	0.91	33	0.29
NGO	0.15	33	0.37
Other	0.02	33	0.14
Local bureaucrat ever filed to be reimbursed for amount spent	0.08	28	0.27
Reason the government doesn't provide 100 percent of the funds			
It is the norm	0.94	29	0.25
They know local bureaucrats earn tips (bribes)	0.90	28	0.30
Philanthropists, NGOs can cover difference	0.70	25	0.47
Hard for government to raise funds through taxing and borrowing	0.27	29	0.45

Notes: Data is from two separate surveys of the local bureaucrats and their supervisors in 2020. Questions were closed ended in both cases.

Table 3: Heterogeneity in sources of funds

	Flood control and relief		Free food to public		Food and logistics during officer visits	
	Mean	N	Mean	N	Mean	N
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Bureaucrat perspective						
Whether local bureaucrats provide service (proportion who agree)	0.61	750	0.25	750	0.82	750
Cost each time (PKR)	-	-	148917	53	59022	612
If a 100 PKR is spent, how much of it is funded through:						
Local bureaucrats' pockets	-	-	52.95	55	83.61	613
Government funds	-	-	8.48	56	0.01	613
Local philanthropists	-	-	31.88	56	9.34	613
NGO	-	-	6.54	56	7.08	613
Other	-	-	0.00	54	0.00	611
Frequency of activities						
Once a year	0.00	449	0.09	187	0.07	617
Twice a year	0.00	449	0.12	187	0.10	617
4 times a year	0.00	449	0.01	187	0.12	617
Every month	0.00	449	0.00	187	0.63	617
Daily	0.01	449	0.77	187	0.00	617
Other (as per requirement)	0.99	449	0.00	187	0.08	617
Panel B: Supervisor perspective						
Whether local bureaucrats provide service (proportion who agree)	0.89	33	0.90	34	0.93	35
Cost each time (PKR)	2406250	8	165182	9	138045	9
If a 100 PKR is spent, how much of it is funded through:						
Local bureaucrats' pockets	12.90	21	15.11	30	81.22	30
Government funds	72.98	21	10.55	30	8.50	30
Local philanthropists	12.82	21	73.13	30	9.11	30
NGO	1.76	21	1.21	30	0.50	30
Other	0.00	21	0.00	30	0.67	30
Frequency of activities						
Once a year	0.58	29	0.45	28	0.09	31
Twice a year	0.06	29	0.12	28	0.08	31
4 times a year	0.00	29	0.09	28	0.16	31
Every month	0.00	29	0.00	28	0.33	31
Other	0.37	29	0.34	28	0.35	31

Notes: Data is from two separate surveys of the local bureaucrats and their supervisors in 2020. Except for questions on costs, the rest were closed ended.

Table 4: Reasons local bureaucrats are willing to spend out of pocket and public goods and services

	Mean	N	SD
	(1)	(2)	(3)
Panel A: Bureaucrat perspective			
Most important reason for spending out of pocket			
If I don't, others in the service will have a bad opinion of me	0.62	613	0.49
It is important for people in my area to receive this good or service	0.30	613	0.46
It is part of my job description	0.01	613	0.12
If I don't, my career service progression would be hurt	0.07	613	0.25
If I don't, I can face disciplinary action	0.00	613	0.00
Other	0.00	613	0.00
Panel B: Supervisor perspective			
Reasons local bureaucrats are willing to spend out of pocket			
If they don't, they can face disciplinary action	0.76	28	0.43
Reduced accountability if local bureaucrats engage in corruption	0.39	28	0.50
If they don't, others in the service will have a bad opinion of them	0.20	28	0.41
It is the norm	0.22	28	0.42
If they don't, their career service progression would be hurt	0.11	28	0.32
It is part of their job description	0.06	28	0.24
Other	0.05	28	0.23
It is important for people in their area to receive this good or service	0.00	28	0.00

Notes: Data is from two separate surveys of the local bureaucrats and their supervisors in 2020. Questions were closed ended in both cases except for the option "Reduced accountability if local bureaucrats engage in corruption", which was volunteered by the supervisors.

A Technical Appendix

A.1 Proofs of results in the text

A.1.1 Retention rule

Proof of Lemma 1. Suppose that the politician receives $s = 1$, then her belief about the bureaucrat's ability is:

$$\begin{aligned}
\mathbb{P}(\omega = 1 \mid s = 1) &= \frac{\mathbb{P}(s = 1 \mid \omega = 1)\mu}{\mathbb{P}(s = 1 \mid \omega = 1)\mu + \mathbb{P}(s = 1 \mid \omega = 0)(1-\mu)} \\
&= \mathbb{P}(\theta = H \mid s = 1)\frac{\phi F(\tau_1 + e_1^*(H))\mu}{\phi F(\tau_1 + e_1^*(H))\mu + \phi F(0)(1-\mu)} \\
&\quad + \mathbb{P}(\theta = D \mid s = 1)\frac{\phi F(\tau_1 + e_1^*(D))\mu}{\phi F(\tau_1 + e_1^*(D))\mu + \phi F(0)(1-\mu)} \\
&= 1
\end{aligned}$$

The payoff from retaining this bureaucrat is therefore $\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)$. Instead, replacing the bureaucrat gives a payoff of $\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$. The politician therefore retains the bureaucrat since:

$$\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) \geq \lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0) > \mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$$

Where the first inequality follows from the fact that $\tau_2^*(r = 1)$ maximizes $\lambda F(\tau_2) - \tau_2$ and the second from the fact that $\mu < 1$.

Suppose instead that the politician receives $s = 0$. Then her belief about the bureaucrat's ability is:

$$\begin{aligned}
\mathbb{P}(\omega = 1 \mid s = 0) &= \frac{\mathbb{P}(s = 0 \mid \omega = 1)\mu}{\mathbb{P}(s = 0 \mid \omega = 1)\mu + \mathbb{P}(s = 0 \mid \omega = 0)(1-\mu)} \\
&= \mathbb{P}(\theta = H \mid s = 0)\frac{(1 - \phi F(\tau_1 + e_1^*(H)))\mu}{(1 - \phi F(\tau_1 + e_1^*(H)))\mu + (1 - \phi F(0))(1-\mu)} \\
&\quad + \mathbb{P}(\theta = D \mid s = 0)\frac{(1 - \phi F(\tau_1 + e_1^*(D)))\mu}{(1 - \phi F(\tau_1 + e_1^*(D)))\mu + (1 - \phi F(0))(1-\mu)}
\end{aligned}$$

This probability is less than μ since, for $\theta \in \{H, D\}$:

$$\frac{(1-\phi F(\tau_1 + e_1^*(\theta)))\mu}{(1-\phi F(\tau_1 + e_1^*(\theta)))\mu + (1-\phi F(0))(1-\mu)} = \frac{1}{1 + \frac{1-\mu}{\mu} \times \frac{1}{1-\phi F(\tau_1 + e_1^*(\theta))}} \leq \mu$$

$$\Leftrightarrow \quad \frac{1-\mu}{\mu} \leq \frac{1-\mu}{\mu} \frac{1}{1-\phi F(\tau_1 + e_1^*(\theta))} \quad \Leftrightarrow \quad 0 \leq \phi F(\tau_1 + e_1^*(\theta))$$

The payoff from retaining this bureaucrat is therefore $\mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)$. Instead, replacing the bureaucrat gives a payoff of $\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$. The politician therefore does not prefer to retain the bureaucrat since:

$$\begin{aligned} \mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) &\leq \mu\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) \\ &\leq \mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0) \end{aligned}$$

Where the first inequality follows from the fact that $\mathbb{P}(\omega = 1 \mid s = 0) \leq \mu$ and the second from the fact that $\tau_2^*(r = 0)$ maximizes $\mu\lambda F(\tau_2) - \tau_2$. The first inequality is strict whenever $\tau_1 + e_1(\theta) > 0$ for some $\theta \in \{H, D\}$. □

A.1.2 Bureaucrat's first period behavior

To prove Lemma 2, we prove the two more general lemmas below. We first define

$$\tau_1 = \begin{cases} f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - c^{-1}(1, D) & \text{if } \phi\mu w_2 f(\Psi) < 1 \\ \Psi - w_1 - c^{-1}(1, D) & \text{if } \phi\mu w_2 f(\Psi) \geq 1, \end{cases}$$

$$\tau_2 = \begin{cases} f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 & \text{if } \phi\mu w_2 f(\Psi) < 1 \\ \Psi - w_1 & \text{if } \phi\mu w_2 f(\Psi) \geq 1 \end{cases} \text{ and } \tau_3 = \begin{cases} f^{-1}\left(\frac{1}{\phi\mu w_2}\right) & \text{if } \phi\mu w_2 f(\Psi) < 1 \\ \Psi & \text{if } \phi\mu w_2 f(\Psi) \geq 1. \end{cases}$$

Using these thresholds, we can state the two Lemmas:

Lemma 4. *Suppose that $\phi\mu w_2 f(\Psi) < 1$,*

- *If $\tau \leq \tau_1$, $e_\theta^*(\tau)$ solves $\mu\phi w_2 f(e + \tau) = c(e - w_1, \theta)$ and $b_\theta^*(\tau) = e_\theta^*(\tau) - w_1$, $\forall \theta \in \{H, D\}$.*
- *If $\tau \in (\tau_1, \tau_2]$, $e_H^*(\tau)$ solves $\mu\phi w_2 f(e_H^*(\tau) + \tau) = c(e_H^*(\tau) - w_1, H)$ and $b_H^*(\tau) = e_H^*(\tau) - w_1$ while $e_D^*(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ and $b_D^*(\tau) = c^{-1}(1, D)$.*
- *If $\tau \in (\tau_2, \tau_3]$, $e_\theta^*(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$, $\forall \theta \in \{H, D\}$, $b_D^*(\tau) = c^{-1}(1, D)$, and $b_H^*(\tau) = 0$.*

- *If* $\tau \geq \tau_3$*,* $e^*_\theta(\tau) = 0,\ \forall\theta \in \{H, D\},\ b^*_D(\tau) = c^{-1}(1, D),\ b^*_H(\tau) = 0.$

Lemma 5. *Suppose that* $\phi\mu w_2 f(\Psi) \geq 1$*,*

- *If* $\tau \leq \tau_1$*, then, for all* $\theta \in \{H, D\}$*,* $e^*_\theta(\tau) = \Psi - \tau$ *if* $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$ *and* $e^*_\theta(\tau)$ *solves* $\mu\phi w_2 f(e + \tau) = c(e - w_1, \theta)$ *otherwise. In both cases,* $b^*_\theta(\tau) = e^*_\theta(\tau) - w_1$*.*

- *If* $\tau \in (\tau_1, \tau_2]$*, then* $e^*_D(\tau) = \Psi - \tau$ *and* $b^*_D = c^{-1}(1, D)$*. Instead,* $e^*_H(\tau) = \Psi - \tau$ *if* $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$ *and* $e^*_H(\tau)$ *solves* $\mu\phi w_2 f(e + \tau) = c(e - w_1, \theta)$ *otherwise with* $b^*_H(\tau) = e^*_H(\tau) - w_1$ *in both cases.*

- *If* $\tau \in (\tau_2, \tau_3]$*, then* $e^*_\theta(\tau) = \Psi - \tau,\ \forall\theta \in \{H, D\}$*,* $b^*_D = c^{-1}(1, D)$ *and* $b^*_H = 0$*.*

- *If* $\tau \geq \tau_3$*,* $e^*_\theta = 0,\ \forall\theta \in \{H, D\}$*,* $b^*_D = c^{-1}(1, D)$ *and* $b^*_H = 0$*.*

Proof of Lemma 4 and 5. Given a tax rate τ and the politician's retention rule from Lemma 1, the bureaucrat's best response solves:

$$\max_{b,e} \quad w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b, \theta) \quad \text{s.t.} \quad 0 \leq e \leq w_1 + b,\ 0 \leq b$$

The Lagrangian is:

$$\mathcal{L}(e, b; \gamma) = w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b, \theta) + \gamma(w_1 + b - e)$$

Where γ is the Lagrange multiplier. The first-order conditions are:

$$\frac{\partial \mathcal{L}(e, b)}{\partial e} = -1 + \mu\phi w_2 f(\tau + e) - \gamma = 0$$
$$\frac{\partial \mathcal{L}(e, b)}{\partial b} = 1 - c(b, \theta) + \gamma = 0$$

The second-order condition is satisfied since F is concave and C is convex (so $-C(b, \theta)$ is concave). There are two cases:

1. **Case 1:** If the constraint does not bind, then by complementary slackness $\gamma = 0$ and the first-order condition with respect to e gives $\mu\phi w_2 f(\tau + e^*_\theta) - 1 = 0$.

   (a) If $\mu\phi w_2 f(\tau) - 1 < 0$, then $\mu\phi w_2 f(\tau + e) - 1 < 0$ for any $e \in [0, \Psi - \tau]$. Since $f(\tau + e) = 0$ for $e > \Psi - \tau$, then $\mu\phi w_2 f(\tau + e) - 1 = -1 < 0$ for $e > \Psi - \tau$. The objective function is therefore everywhere decreasing in e and the unconstrained optimal is $e^* = 0$. Given $e^* = 0$, the constraint indeed does not bind, so in this case $e^* = 0$ is also the constrained optimal.

(b) If $\mu\phi w_2 f(\Psi) - 1 > 0$, then $\mu\phi w_2 f(\tau + e) - 1 > 0$ for any $e \in [0, \Psi - \tau]$, so the first-order condition can never be satisfied since $\mu\phi w_2 f(\tau + e) - 1$ is either strictly greater than zero, if $e \leq \Psi - \tau$, or strictly less than zero, if $e > \Psi - \tau$ (as $\mu\phi w_2 f(\tau + e) - 1 = -1 < 0$ for $\tau + e > \Psi$). In this case, the objective function is strictly increasing in e for any $e \leq \Psi - \tau$ and strictly decreasing in e for any $e > \Psi - \tau$, so the unconstrained optimal is $e^* = \Psi - \tau$.

(c) If $\mu\phi w_2 f(\tau) - 1 > 0$ but $\mu\phi w_2 f(\Psi) - 1 \leq 0$, then the first-order condition is satisfied for some $e^* \in [0, \Psi - \tau]$ such that $\mu\phi w_2 f(\tau + e^*) = 1$. The unconstrained optimal is therefore $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$.

We now turn to solving for the unconstrained optimal bribe level in order to characterize when the budget constraint binds and to determine whether the unconstrained optima above are also constrained optima. If the budget constraint is not binding ($\gamma = 0$), the first-order condition with respect to b gives $c(b_D^*, D) = 1$ for type D but is never satisfied for type H since $c(b, H) > c(0, H) = 1$ for any $b > 0$ (by convexity of C). The budget constraint is therefore binding if $e^* \geq w_1 + c^{-1}(1, D)$ for $\theta = D$ and if $e^* \geq w_1$ for $\theta = H$. We can then solve for the constrained optima:

(a) If $\mu\phi w_2 f(\tau) - 1 < 0$, the constraint never binds so the constrained optimal personal funding is $e_\theta^*(\tau) = 0$ as described above.

(b) If $\mu\phi w_2 f(\Psi) - 1 > 0$, then the unconstrained optimal private funding is $e^* = \Psi - \tau$, so the budget constraint is satisfied if $\Psi - \tau < w_1$ for type $\theta = H$ and if $\Psi - \tau < w_1 + c^{-1}(1, D)$ for type $\theta = D$. When these constraints are satisfied, the constrained optimal personal funding is therefore $e_\theta^*(\tau) = \Psi - \tau$.

(c) If $\mu\phi w_2 f(\tau) - 1 \geq 0$ but $\mu\phi w_2 f(\Psi) - 1 \leq 0$, then the unconstrained optimal private funding is $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$, so the budget constraint is satisfied if $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau < w_1$ for $\theta = H$, and $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau < w_1 + c^{-1}(1, D)$ for $\theta = D$. When these constraints are satisfied, the constrained optimal personal funding is therefore $e_\theta^*(\tau) = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$.

2. **Case 2:** If any of the solutions above violate the budget constraint, then the budget constraint must bind at the optimal level of funding and bribe, so $\gamma > 0$. We can substitute the bribe into the bureaucrat's problem by using the binding constraint:

$e = w_1 + b$ or, equivalently, $b = e - w_1$. Substituting in the first-order conditions and solving them simultaneously gives

$$\mu\phi w_2 f(\tau + e) = 1 + \gamma = c(e - w_1, \theta)$$

(a) If $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$, then the objective function is increasing for any $e \in [0, \Psi - \tau]$ even with the constraint binding so type θ chooses the highest possible funding level, $e^*_\theta(\tau) = \Psi - \tau$.

(b) If $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, \theta)$, then we use the intermediate value theorem to show that there exists a value of e that solves $\mu\phi w_2 f(\tau + e) = c(e - w_1, \theta)$. Let $LHS(e) = \mu\phi w_2 f(e + \tau)$ and $RHS(e) = c(e - w_1, \theta)$. First note that $LHS(e)$ is decreasing in e since f is decreasing and $RHS(e)$ is increasing in e since c is increasing. We therefore need to show that $LHS(e) > RHS(e)$ at the smallest value of e and $LHS(e) < RHS(e)$ at the largest value of e. We consider two cases depending on whether the maximum value of funding is attained.

  i. Suppose first that $\mu\phi w_2 f(\Psi) - 1 > 0$. In this case, the largest possible value of e is the unconstrained optimal $e = \Psi - \tau$. At this value of e, $LHS(e) = \mu\phi w_2 f(\Psi)$ and $RHS(e) = c(\Psi - \tau - w_1, \theta)$. Since we are looking at the case where $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, \theta)$, then $LHS(\Psi - \tau) < RHS(\Psi - \tau)$. At the smallest value of e such that the constraint binds, we can show that $LHS(e) > RHS(e)$ for both $\theta \in \{H, D\}$. We consider the two types in turns. For $\theta = H$, the lowest value of e such that the constraint binds is $e = w_1$. At $e = w_1$, we have that, $\forall \tau \in [0, \Psi - w_1]$, $LHS(e) = \mu\phi w_2 f(w_1 + \tau) > \mu\phi w_2 f(w_1 + \Psi - w_1) = \mu\phi w_2 f(\Psi) > 1 = c(0, H) = RHS(e)$ where the last inequality follows from the fact that $\mu\phi w_2 f(\Psi) - 1 > 0$. For $\theta = D$, the lowest value of e such that the constraint binds is $e = w_1 + c^{-1}(1, D)$. At $e = w_1 + c^{-1}(1, D)$, we have that, $\forall \tau \in \left[0, \Psi - w_1 - c^{-1}(1, D)\right]$, $LHS(e) = \mu\phi w_2 f(w_1 + c^{-1}(1, D) + \tau) > \mu\phi w_2 f(w_1 + c^{-1}(1, D) + \Psi - w_1 - c^{-1}(1, D)) = \mu\phi w_2 f(\Psi) > 1 = c(c^{-1}(1, D), D) = c(w_1 + c^{-1}(1, D) - w_1, D) = RHS(e)$ where the last inequality follows from the fact that $\mu\phi w_2 f(\Psi) - 1 > 0$. Therefore, since $LHS(e)$ is decreasing in e and $RHS(e)$ is increasing in e, $LHS(e) > RHS(e)$ at the smallest value of e and $LHS(e) < RHS(e)$ at the largest value of e, then by the intermediate value theorem, there exists $e^*_\theta(\tau) \in [w_1 + \mathbb{1}\{\theta = D\} c^{-1}(1, D), \Psi - \tau]$ such that $LHS(e^*_\theta(\tau)) = RHS(e^*_\theta(\tau))$.

ii. Consider now the case where $\mu\phi w_2 f(\Psi) - 1 \leq 0$. In this case, the largest possible value of e is the unconstrained optimal $e = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. At this value of e, $LHS(e) = \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau + \tau\right) = 1$ and $RHS(e) = c(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1, \theta)$. For type $\theta = H$, we have that $c(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1, H) > c(0, H) = 1$ since c is increasing and since $c(0, H) = 1$, so $RHS(e) > LHS(e)$. Similarly, for type $\theta = D$, we have $c(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1, D) > 1$. This follows from the fact that the constraint is binding at $e = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$, so that $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau > w_1 + c^{-1}(1, D)$, which is equivalent to $c(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1, D) > 1$. At the smallest value of e such that the constraint binds, we can show that $LHS(e) > RHS(e)$ for both $\theta \in \{H, D\}$. We consider the two types in turns. For $\theta = H$, the lowest value of e such that the constraint binds is $e = w_1$. At $e = w_1$, we have that, $\forall \tau \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1\right]$, $LHS(e) = \mu\phi w_2 f(w_1 + \tau) > \mu\phi w_2 f\left(w_1 + f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1\right) = \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) = 1 = c(0, H) = RHS(e)$. For $\theta = D$, the lowest value of e such that the constraint binds is $e = w_1 + c^{-1}(1, D)$. At $e = w_1 + c^{-1}(1, D)$, we have that, $\forall \tau \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - c^{-1}(1, D)\right]$, $LHS(e) = \mu\phi w_2 f\left(w_1 + c^{-1}(1, D) + \tau\right) > \mu\phi w_2 f\left(w_1 + c^{-1}(1, D) + f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - c^{-1}(1, D)\right) = \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) = 1 = c(c^{-1}(1, D), D) = c(w_1 + c^{-1}(1, D) - w_1, D) = RHS(e)$. Therefore, since $LHS(e)$ is decreasing in e and $RHS(e)$ is increasing in e, $LHS(e) > RHS(e)$ at the smallest value of e and $LHS(e) < RHS(e)$ at the largest value of e, then by the intermediate value theorem, there exists $e^*_\theta(\tau) \in [w_1 + \mathbb{1}\{\theta = D\}c^{-1}(1, D), \Psi - \tau]$ such that $LHS(e^*_\theta(\tau)) = RHS(e^*_\theta(\tau))$.

Finally, we map these results to the different cases in Lemmas 4 and 5. The four possible cases in Lemma 5 correspond to all the cases where $\mu\phi w_2 f(\Psi) - 1 > 0$ above:

1. If $\tau \leq \Psi - w_1 - c^{-1}(1, D) = \tau_1$, the budget constraint of both types binds and the solution falls under either case 2(a) or case 2(b)i above. For each $\theta \in \{H, D\}$, if $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$ (that is, if $\tau \geq \Psi - w_1 - c^{-1}(\mu\phi w_2 f(\Psi), \theta)$), then $e^*_\theta(\tau) = \Psi - \tau$. Otherwise, $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e + \tau) = c(e - w_1, \theta)$ and $b^*_\theta(\tau) = e^*_\theta(\tau) - w_1$.

2. If $\tau \in \left(\Psi - w_1 - c^{-1}(1, D), \Psi - w_1\right]$, i.e., $\tau \in (\tau_1, \tau_2]$, the budget constraint of the honest type binds but not that of the dishonest type. So the honest type falls under case 2(a) or 2(b)i above but the dishonest type falls under case 1(b). The honest type's private funding and bribe solve $e^*_H(\tau) = \Psi - \tau$ if $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$ and

$\mu\phi w_2 f(e_H^*(\tau)+\tau) = c(e_H^*(\tau)-w_1, H)$ if $\mu\phi w_2 f(\Psi) < c(\Psi-\tau-w_1,\theta)$. In both cases, $b_H^*(\tau) = e_H^*(\tau) - w_1$. The dishonest type's funding and bribe are: $e_D^*(\tau) = \Psi - \tau$ and $b_D^*(\tau) = c^{-1}(1, D)$.

3. If $\tau \in (\Psi - w_1, \Psi]$, i.e., $\tau \in (\tau_2, \tau_3]$, neither types' budget constraint binds so both fall under case 1(b) above: $e_\theta^*(\tau) = \Psi - \tau$, $b_\theta^*(D) = c^{-1}(1, D)$, and $b_\theta^*(H) = 0$.

4. If $\tau \geq \Psi = \tau_3$, then $f(\tau + e) = 0$ for any $e \in [0, +\infty)$, so $\mu\phi w_2 f(\tau+e) - 1 = -1 < 0$. Therefore both types choose $e_\theta^*(\tau) = 0,\ b_\theta^*(D) = c^{-1}(1, D),\ b_\theta^*(H) = 0$.

The four possible cases in Lemma 4 correspond to all the cases where $\mu\phi w_2 f(\Psi) - 1 < 0$:

1. If $\tau \leq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - c^{-1}(1, D) = \tau_1$, the budget constraint of both types binds. For type $\theta = H$, $\mu\phi w_2 f(\Psi) - 1 < 0$ implies $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, H)$ since $c(e, H) \geq 1$ so the solution falls under case 2(b)ii above: $e_H^*(\tau)$ solves $\mu\phi w_2 f(e+\tau) = c(e - w_1, H)$ and $b_H^*(\tau) = e_H^*(\tau) - w_1$. For type $\theta = D$, the solution either falls under case 2(a) or 2(b)ii above. If $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, D)$, then $e_D^*(\tau) = \Psi - \tau$. Otherwise, $e_D^*(\tau)$ solves $\mu\phi w_2 f(e+\tau) = c(e - w_1, D)$ and $b_D^*(\tau) = e_D^*(\tau) - w_1$.

2. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - c^{-1}(1, D), f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right]$, i.e., $\tau \in (\tau_1, \tau_2]$, the budget constraint of the honest type binds but not that of the dishonest type. So the honest type falls under case 2(b)ii above but the dishonest type falls under case 1(c). The honest type's private funding and bribe solve $\mu\phi w_2 f(e_H^* + \tau) = c(e_H^* - w_1, H)$ and $b_H^*(\tau) = e_H^*(\tau) - w_1$. The dishonest type's funding and bribe are: $e_D^*(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ and $b_D^*(\tau) = c^{-1}(1, D)$.

3. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1, f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right]$, i.e., $\tau \in (\tau_2, \tau_3]$, neither types' budget constraint binds so both fall under case 1(c) above: $e_\theta^*(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$, $b_D^*(\tau) = c^{-1}(1, D)$ and $b_H^*(\tau) = 0$.

4. If $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) = \tau_3$, then $\mu\phi w_2 f(\tau) < 1$ so case 1(a) applies: $e_\theta^*(\tau) = 0,\ b_\theta^*(D) = c^{-1}(1, D),\ b_\theta^*(H) = 0$

Finally, we can rule out that there exist any other class of equilibria than those leading to the three types of policies listed in Section 5.3. First note that, since the bureaucrat is not aware of his own ability, his choice of funding and bribes cannot be conditioned on ability. As a result, all equilibria must be pooling equilibria in which all ability types choose the same level of funding and bribes (i.e., $b^*(\omega,\theta) = b^*(\omega',\theta)$, $\forall \omega,\omega' \in \{0,1\}$ and $e^*(\omega,\theta) = e^*(\omega',\theta)$, $\forall \omega,\omega' \in \{0,1\}$).

We can first rule out any equilibria in which only one of the two honesty types (θ) contributes no funding. Suppose that, for a given τ, neither type of bureaucrat's budget constraint is binding. When the budget constraint is not binding, the bribes do not matter for the choice of funding, so the marginal benefit and marginal cost of additional funding is the same for both types. Since the optimal unconstrained funding is $e^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$, then if one type of bureaucrat finds it optimal not to contribute, it must be that the tax level is such that $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, so the other type's optimal funding would also be $e^* = 0$. Since $w_1 > 0$, the budget constraint is indeed not binding when $e^* = 0$ so the optimal funding is indeed independent of type. Second, we can rule out equilibria in which the honest bureaucrat takes bribes but does not contribute. This follows directly from the fact that $c(0,H) = 1$. Indeed, if the honest bureaucrat does not contribute, $e^*_H = 0$, then the bureaucrat's budget constraint is not binding since $0 < w_1$. If the bureaucrat's budget constraint is not binding, then he chooses bribes b to maximize: $w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b,H)$ subject to $0 \leq b$. The objective function's derivative with respect to b is negative for any $b \geq 0$ since $1 - c(b,H) < 0$ given $c(0,H) = 1$ and that $C(\cdot)$ is convex.

Finally, we can rule out equilibria in which the dishonest bureaucrat takes no bribes. Suppose first that the dishonest bureaucrat's budget constraint is not binding: $e^* < w_1$. In this case, the bureaucrat chooses bribes b to maximize: $w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b,D)$. The objective function's derivative, evaluated at $b = 0$ is $1 - c(0,D) > 1 - c(0,H) = 0$. Therefore, it cannot be optimal for the dishonest bureaucrat to set $b = 0$ when the budget constraint is not binding. Suppose instead that the budget constraint is binding, then the first-order conditions on e and b combined with the binding budget constraint, $e = w_1 + b$, imply that $\mu\phi w_2 f(\tau + w_1 + b) = c(b,D)$. If $b = 0$, then this condition becomes $\mu\phi w_2 f(\tau + w_1) = c(0,D)$. Since the constraint binds when the unconstrained optimal funding, $e^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ is greater than $w_1 + b$, then when $b = 0$, a binding constraint implies $f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau > w_1$, or equivalently $\mu\phi w_2 f(\tau + w_1) > 1$. Since $c(0,D) < c(0,H) = 1$, then $\mu\phi w_2 f(\tau + w_1) > 1 > c(0,D)$ so the first-order conditions $\mu\phi w_2 f(\tau + w_1 + b) = c(b,D)$ cannot be satisfied at $b = 0$ and we must have $b > 0$.

As a result, the only possible equilibrium outcomes are the ones covered by the three types of policies we define. □

A.1.3 Politician's first period behavior

Proof of Lemma 3. Using Lemmas 4 and 5, we can substitute the bureaucrat's best-response into the politician's expected payoff. We begin by simplifying this expected payoff by substituting the second-period tax level:

Claim: Given assumption 2, $\tau_2^*(r = 1) = \tau_2^*(r = 0) = \Psi$.

Proof. If the bureaucrat is retained, he is high-ability, so the second-period objective function is $\lambda F(\tau) - \tau$. The derivative of that function is $\lambda f(\tau) - 1$ for any $\tau \in [0, \Psi]$ and -1 for any $\tau > \Psi$. Given assumption 2, $\mu < 1$, and that f is decreasing, we have $\lambda f(\tau) - 1 > \mu\lambda f(\Psi) - 1 > 0$ for any $\tau \in [0, \Psi]$. Therefore, $\lambda F(\tau) - \tau$ is maximized at $\tau_2^*(r = 1) = \Psi$. If the bureaucrat is not retained, the second-period objective function is $\mu\lambda F(\tau) - \tau$. The derivative of that function is $\mu\lambda f(\tau) - 1$ for any $\tau \in [0, \Psi]$ and -1 for any $\tau > \Psi$. Given assumption 2 and that f is decreasing, we have $\mu\lambda f(\tau) - 1 \geq \mu\lambda f(\Psi) - 1 > 0$ for any $\tau \in [0, \Psi]$. Therefore, $\mu\lambda F(\tau) - \tau$ is maximized at $\tau_2^*(r = 0) = \Psi$. □

Therefore, the second-period expected payoffs are $\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) = \lambda - \Psi$ when a high-ability bureaucrat is retained and $\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0) = \mu\lambda - \Psi$ when a new bureaucrat is drawn from the pool. We next proceed in three steps.

Step 1: First, we derive the slope of the first segment of the function $V(\tau)$ (when $\tau \in [0, \tau_2])$ for different values of ϕ.

CASE 1: When $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, $\tau_2 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 \leq 0$, so there is no value of τ for which the honest bureaucrat takes additional bribes and the informal policy with high corruption can never happen. In this case, we define $\underline{\nu} = 1$ so that $\nu \leq \underline{\nu}$, $\forall \nu \in [0, 1]$.

CASE 2: When $\frac{1}{\mu w_2 f(w_1)} < \phi < \frac{1}{\mu w_2 f(\Psi)}$, then $\tau_2 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 > 0$ and $\phi\mu w_2 f(\Psi) - 1 < 0$ so Lemma 4 applies. For $\tau \in [0, \tau_2]$, $e_H^*(\tau)$ solves $\mu\phi w_2 f(e_H^*(\tau) + \tau) = c(e_H^*(\tau) - w_1, H)$ and $b_H^*(\tau) = e_H^*(\tau) - w_1$ while $e_D^*(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ and $b_D^*(\tau) = c^{-1}(1, D)$. Abusing notation and denoting $\mu(\omega) = \mathbb{P}(\omega)$ and $\nu(\theta) = \mathbb{P}(\theta)$, the expected intertemporal payoff becomes:

$$
\begin{aligned}
V(\tau) = &\sum_{\omega\in\{0,1\}}\sum_{\theta\in\{H,D\}} \mu(\omega)\nu(\theta)\Big[\lambda F(\omega(\tau+e_\theta^*(\tau))) - \tau - \eta b_\theta^*(\tau) + \phi F(\omega(\tau+e_\theta^*(\tau)))(\lambda-\Psi) \\
&\qquad + (1-\phi F(\omega(\tau+e_\theta^*(\tau))))(\mu\lambda-\Psi)\Big] \\
= &\ \nu\left[\mu F(\tau+e_D^*(\tau))\left(\lambda+\phi((\lambda-\Psi)-(\mu\lambda-\Psi))\right)+\mu\lambda-\Psi-\eta b_D^*(\tau)-\tau\right] \\
&\quad + (1-\nu)\left[F(\tau+e_H^*(\tau))\left(\lambda+\phi((\lambda-\Psi)-(\mu\lambda-\Psi))\right)+\mu\lambda-\Psi-\eta b_H^*(\tau)-\tau\right] \\
= &\ \nu\left[\mu F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\left(\lambda+\phi\lambda(1-\mu)\right)-\eta c^{-1}(1,D)-\tau\right] \\
&\quad + (1-\nu)\left[\mu F(\tau+e_H^*(\tau))\left(\lambda+\phi\lambda(1-\mu)\right)-\eta(e_H^*(\tau)-w_1)-\tau\right]+\mu\lambda-\Psi
\end{aligned}
$$

With $U_2 := \lambda + \phi\lambda(1-\mu)$, the derivative of $V(\tau)$ with respect to τ for $\tau \in [0, \tau_2]$ is:

$$
\frac{\partial V(\tau)}{\partial \tau} = \nu(-1) + (1-\nu)\left[\mu U_2 f(\tau+e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right)-\eta\frac{\partial e_H^*(\tau)}{\partial\tau}-1\right]
$$

This derivative is positive if and only if:

$$
\mu U_2 f(\tau+e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right)-\eta\frac{\partial e_H^*(\tau)}{\partial\tau}-1 \geq \nu\left(\mu U_2 f(\tau+e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right)-\eta\frac{\partial e_H^*(\tau)}{\partial\tau}\right)
$$

Next, we show that $\mu U_2 f(\tau+e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right)-\eta\frac{\partial e_H^*(\tau)}{\partial\tau}-1>0$. We first note the following result:

Lemma 6. *For any $\tau \in [0,\tau_2]$, $1+\frac{\partial e_H^*(\tau)}{\partial\tau}>0$, where $e_H^*(\tau)$ is as characterized in Lemma 4.*

Proof of Lemma 6. From Lemma 4, we know that when $\tau\in[0,\tau_2]$, $e_H^*(\tau)$ solves $\mu\phi w_2 f(e_H^*(\tau)+\tau) = c(e_H^*(\tau)-w_1,H)$. Differentiating both sides with respect to τ gives:

$$
\mu\phi w_2 f'(e_H^*(\tau)+\tau)\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right) = c'(e_H^*(\tau)-w_1,H)\frac{\partial e_H^*(\tau)}{\partial\tau}
$$

Therefore, $1+\frac{\partial e_H^*(\tau)}{\partial\tau} = \frac{c'(e_H^*(\tau)-w_1,H)\frac{\partial e_H^*(\tau)}{\partial\tau}}{\mu\phi f'(e_H^*(\tau)+\tau)} > 0$ since $c'(\cdot)>0$, $f'(\cdot)<0$ (by strict concavity of F) and $\frac{\partial e_H^*(\tau)}{\partial\tau}<0$. We can therefore conclude that $1+\frac{\partial e_H^*(\tau)}{\partial\tau}>0$ for any $\tau\in[0,\tau_2]$. □

Since F is strictly increasing on $[0,\Psi]$, we know that $f(\tau+e_H^*(\tau))>0$ for any $\tau\in[0,\tau_2]$. Therefore, given Lemma 6, $\mu U_2 f(\tau+e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial\tau}\right)>0$ for any $\tau\in[0,\tau_2]$. Finally, note that by assumption 2, $\mu U_2 f(e_H^*(\tau)+\tau)\geq\mu\lambda f(\Psi)>1$ for any $e_H^*(\tau)+\tau\leq\Psi$. Moreover, since

$\eta > 1$ and $\frac{\partial e^*_H(\tau)}{\partial\tau} < 0$, then $1 + \frac{\partial e^*_H(\tau)}{\partial\tau} > 1 + \eta\frac{\partial e^*_H(\tau)}{\partial\tau}$. Therefore, $\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) > 1 \times \left(1 + \eta\frac{\partial e^*_H(\tau)}{\partial\tau}\right)$, which implies $\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} - 1 > 0$ for any $\tau \in [0, \tau_2]$. Finally, since $\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} > \mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} - 1$, this implies that $\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} > 0$ for any $\tau \in [0, \tau_2]$. Therefore, in this case, we can define:

$$\bar{\nu} = \max_{\tau\in[0,\tau_2]}\left\{\frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau}}\right\} \tag{1}$$

$$\underline{\nu} = \min_{\tau\in[0,\tau_2]}\left\{\frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial\tau}}\right\} \tag{2}$$

We have that (1) $\bar{\nu} \in (0, 1)$ and $\underline{\nu} \in (0, 1)$ and (2) $V(\tau)$ is increasing if $\nu \le \underline{\nu}$ and decreasing if $\nu \ge \bar{\nu}$.

CASE 3: When $\phi \ge \frac{1}{\mu w_2 f(\Psi)}$, then $\tau_2 = \Psi - w_1 > 0$ and $\phi\mu w_2 f(\Psi) - 1 \ge 0$ so Lemma 5 applies. For $\tau \in [0, \tau_2]$, $e^*_H(\tau)$ solves $\mu\phi w_2 f(e^*_H(\tau) + \tau) = c(e^*_H(\tau) - w_1, H)$ if $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, H)$ and $e^*_H(\tau) = \Psi - \tau$ if $\mu\phi w_2 f(\Psi) \ge c(\Psi - \tau - w_1, H)$ with $b^*_H(\tau) = e^*_H(\tau) - w_1$ in both cases, while $e^*_D(\tau) = \Psi - \tau$ and $b^*_D(\tau) = c^{-1}(1, D)$. The expected intertemporal payoff becomes:

$$V(\tau) = \begin{cases} \mu U_2\left[\nu F(\Psi - \tau + \tau) + (1-\nu)F(\tau + e^*_H(\tau))\right] & \\ \quad - (1-\nu)\eta(e^*_H(\tau) - w_1) - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi & \text{if } \mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, H) \\ \mu U_2 F(\Psi - \tau + \tau) - (1-\nu)\eta(\Psi - \tau - w_1) & \\ \quad - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi & \text{if } \mu\phi w_2 f(\Psi) \ge c(\Psi - \tau - w_1, H) \end{cases}$$

Using $F(\Psi) = 1$, this becomes:

$$V(\tau) = \begin{cases} \mu U_2\left[\nu + (1-\nu)F(\tau + e^*_H(\tau))\right] & \\ \quad - (1-\nu)\eta(e^*_H(\tau) - w_1) - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi & \text{if } \mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, H) \\ \mu U_2 - (1-\nu)\eta(\Psi - \tau - w_1) - \nu\eta c^{-1}(1, D) & \\ \quad - \tau + \mu\lambda - \Psi & \text{if } \mu\phi w_2 f(\Psi) \ge c(\Psi - \tau - w_1, H) \end{cases}$$

- If $\mu\phi w_2 f(\Psi) \ge c(\Psi - w_1, H)$, then $\mu\phi w_2 f(\Psi) \ge c(\Psi - \tau - w_1, H)$ for any $\tau \in [0, \tau_2]$, so the derivative of $V(\tau)$ with respect to τ is: $\frac{\partial V(\tau)}{\partial\tau} = -(1 - (1-\nu)\eta)$. This is positive if and only if $1 \leq \eta - \nu\eta$ or equivalently $\nu \leq \frac{\eta-1}{\eta}$. Finally, note that when $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, H)$, $e^*_H = \Psi - \tau$, so $\frac{\partial e^*_H(\tau)}{\partial \tau} = -1$. Therefore, we can also denote the threshold as $\bar{\nu}$ and $\underline{\nu}$ since in this case:

$$\bar{\nu} = \underline{\nu} = \frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}} = \frac{\eta - 1}{\eta}$$

- If $\mu\phi w_2 f(\Psi) < c(\Psi - w_1, H)$, then there exists some $\tilde{\tau} \in [0, \tau_2]$ such that $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, H)$ if $\tau < \tilde{\tau}$ and $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, H)$ if $\tau \geq \tilde{\tau}$. The derivative of $V(\tau)$ with respect to τ is then:

$$\frac{\partial V(\tau)}{\partial \tau} = \begin{cases} \nu(-1) + (1-\nu)\left[\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1\right] & \text{if } \tau < \tilde{\tau} \\ -(1 - (1-\nu)\eta) & \text{if } \tau \geq \tilde{\tau} \end{cases}$$

  Following the same logic as in Case 2, this is positive if $\nu \leq \underline{\nu}$ and negative if $\nu \geq \bar{\nu}$ where the thresholds are defined as:

$$\bar{\nu} = \max\left\{ \max_{\tau \in [0, \tau_2]} \left\{ \frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}} \right\}, \frac{\eta - 1}{\eta} \right\} \quad (3)$$

$$\underline{\nu} = \min\left\{ \min_{\tau \in [0, \tau_2]} \left\{ \frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}} \right\}, \frac{\eta - 1}{\eta} \right\} \quad (4)$$

Therefore, we conclude that in all cases, the first segment is increasing if $\nu \leq \underline{\nu}$ and decreasing if $\nu \geq \bar{\nu}$.

Step 2: Second, we show that the slope of the second segment of the function $V(\tau)$, (when $\tau \in [\tau_2, \tau_3]$) is negative.

1. If $\frac{1}{\mu w_2 f(w_1)} < \phi < \frac{1}{\mu w_2 f(\Psi)}$, the function is equal to $V(\tau) = \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi$. The derivative with respect to τ is $\frac{\partial V(\tau)}{\partial \tau} = -1 < 0$.
2. If $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$, the function is equal to $V(\tau) = \mu U_2 F(\Psi) - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi$. The derivative with respect to τ is $\frac{\partial V(\tau)}{\partial \tau} = -1 < 0$.

Step 3: Finally, we note that the function $V(\tau)$ is continuous at $\tau = \tau_2$. To see this, first note that $V(\tau)$ is a continuous function of $b^*_\theta(\tau)$ and $e^*_\theta(\tau)$. Second note that, since the bureaucrat's objective function $U(b, e \mid \tau) = w_1 + b - e + \mu\phi w_2 F(\tau + e) - C(b, \theta)$ is continuous in e, b, and τ, then by Berge's theorem of the maximum, the maximizers $b^*_\theta(\tau)$ and $e^*_\theta(\tau)$ are continuous functions of τ (since the maximizers are single-valued). Therefore, $V(\tau)$ is a composition of continous functions and is therefore continuous everywhere, including at $\tau = \tau_2$.

Therefore, we can conclude from steps 1 to 3 that,

1. When $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, the only possible informal policy is an informal policy with low corruption. Since we defined $\underline{\nu} = 1$ in this case, then $\nu \leq \underline{\nu}$ for any $\nu \in (0, 1)$ so the politician indeed prefers an informal policy with low corruption if $\nu \leq \underline{\nu}$.

2. When $\frac{1}{\mu w_2 f(w_1)} < \phi$, we defined $\bar{\nu}$ and $\underline{\nu}$ as per expressions (1) and (2) or expressions (3) and (4).

   (a) If $\nu \leq \underline{\nu}$, the first segment is monotonically increasing, so $V(\tau) \leq V(\tau_2)$ for any $\tau \in [0, \tau_2]$ and the second segment is decreasing, so $V(\tau_2) \geq V(\tau)$ for any $\tau \in [\tau_2, \tau_3]$. Therefore, the first two segments are maximized at $\tau = \tau_2$ on $[0, \tau_3]$, which corresponds to an informal policy with low corruption.

   (b) If $\nu \geq \bar{\nu}$, the first segment is monotonically decreasing, so $V(0) \geq V(\tau)$ for any $\tau \in [0, \tau_2]$ and the second segment is decreasing, so $V(\tau_2) \geq V(\tau)$ for any $\tau \in [\tau_2, \tau_3]$. Therefore, the first two segments are maximized at $\tau = 0$ on $[0, \tau_3]$, which corresponds to an informal policy with high corruption.

Finally, it is straightforward to show that in both cases, $\bar{\nu}$ and $\underline{\nu}$ are increasing in η. Indeed,

$$\frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}} = 1 - \frac{1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}}$$

and $\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}$ is increasing in η since $\frac{\partial e^*_H(\tau)}{\partial \tau} < 0$. Since $\frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta \frac{\partial e^*_H(\tau)}{\partial \tau}}$ is increasing in η for each τ then the maximum and the minimum of that function are also increasing in η.

□

Proof of Proposition 1. Suppose that $\phi \geq \frac{1}{\mu w_2 f(w_1)}$ and that, for any $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$, $\nu > \bar{\nu}$. The proof proceeds in two parts. We first derive the optimal tax rate on each segment, the maximum value of each segment, and the condition for the informal policy to be preferred to the formal policy. In the second part, we show that there exists a unique threshold on ϕ for this condition to be satisfied.

Part 1: There are two cases to consider. When $\frac{1}{\mu w_2 f(w_1)} \leq \phi < \frac{1}{\mu w_2 f(\Psi)}$, then $\tau_2 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 > 0$ and $\phi\mu w_2 f(\Psi) - 1 < 0$ so Lemma 4 applies. When $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$, $\phi\mu w_2 f(\Psi) - 1 \geq 0$ so Lemma 5 applies.

1. **CASE 1:** $\frac{1}{\mu w_2 f(w_1)} \leq \phi < \frac{1}{\mu w_2 f(\Psi)}$. Using Lemma 4 we can substitute the bureaucrat's best-response into the politician's problem. Recall that we defined $U_2 = \lambda + \phi\lambda(1-\mu)$. The politician's problem becomes:

$$\max_{\tau\in[0,+\infty)} V(\tau) = \begin{cases} \nu\left[\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \eta c^{-1}(1,D)\right] & \\ +(1-\nu)\left[\mu U_2 F(\tau + e_H^*(\tau)) - \eta(e_H^*(\tau) - w_1)\right] - \tau + \mu\lambda - \Psi & \text{if } \tau \in [0,\tau_2] \\ \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1,D) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [\tau_2,\tau_3] \\ \mu U_2 F(\tau) - \tau - \nu\eta c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \in [\tau_3,\Psi] \\ \mu U_2 - \tau - \nu\eta c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \geq \Psi \end{cases}$$

   To solve this problem, we maximize each section of the function piece-wise and then compare the maximum payoff on each section.

   (a) For $\tau \in [0,\tau_2]$, we know from Lemma 3 that when $\nu > \bar{\nu}$, the first segment is decreasing in τ. Therefore the first segment is maximized at $\tau = 0$. If $\nu > \bar{\nu}$, the maximum of this segment is therefore:

$$V(0) = \nu\left[\mu F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)U_2 - \eta c^{-1}(1,D)\right] + (1-\nu)\left[\mu F(e_H^*(0))U_2 - \eta(e_H^*(0) - w_1)\right] + \mu\lambda - \Psi$$

   (b) When $\tau \in [\tau_2,\tau_3]$, the derivative of the payoff function with respect to τ is $\frac{\partial V(\tau)}{\partial\tau} = -1$. This segment is therefore maximized at $\tau = \tau_2$. However, we showed that $V(0) > V(\tau_2)$, so it is never optimal to set the tax in $[\tau_2,\tau_3]$.

   (c) When $\tau \in [\tau_3,\Psi]$, the derivative of the payoff function with respect to τ is

$\frac{\partial V(\tau)}{\partial \tau} = \mu f(\tau)U_2 - 1$. The optimal tax level is $\tau = \Psi$ since for any $\tau \in [0, \Psi]$, $\mu f(\tau)U_2 - 1 = \mu f(\tau)\lambda + \mu f(\tau)\lambda(1-\mu) - 1 > \mu f(\tau)\lambda - 1 > 0$ (by assumption 2). The third segment of the function $V(\tau)$ is therefore increasing everywhere on $\tau \in [\tau_3, \Psi]$. The maximum of this segment is therefore:

$$V(\Psi) = \mu U_2 - \Psi - \nu\eta c^{-1}(1, D) + \mu\lambda - \Psi$$

(d) When $\tau \geq \Psi$, the derivative of the payoff function with respect to τ is $\frac{\partial V(\tau)}{\partial \tau} = -1$ so the optimal tax level is $\tau = \Psi$. The maximum of this segment is therefore also: $V(\Psi) = \mu U_2 - \Psi - \nu\eta c^{-1}(1, D) + \mu\lambda - \Psi$.

To find the global maximizer, we therefore need to compare $V(0)$ to $V(\Psi)$. When $\frac{1}{\mu w_2 f(w_1)} \leq \phi < \frac{1}{\mu w_2 f(\Psi)}$, the politician chooses an informal policy ($\tau = 0$ and $e > 0$) if $V(0) > V(\Psi)$, that is if:

$$\begin{aligned} & \nu\left[\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \eta c^{-1}(1, D)\right] + (1-\nu)\left[\mu U_2 F(e_H^*(0)) - \eta(e_H^*(0) - w_1)\right] \\ & \quad + \mu\lambda - \Psi > \mu U_2 F(\Psi) - \Psi - \nu\eta c^{-1}(1, D) + \mu\lambda - \Psi \\ \Leftrightarrow \quad & \nu\left[\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] + (1-\nu)\left[\mu U_2 F(e_H^*(0)) - \eta(e_H^*(0) - w_1)\right] > \mu U_2 - \Psi \end{aligned} \tag{5}$$

2. **CASE 2:** $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$. In this case, recall from Lemma 5 that the condition $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$ determines whether the honest bureaucrat provides the maximum possible private funding $\Psi - \tau$ or an interior level of funding $e_H(\tau)$ that solves $\mu\phi w_2 f(e + \tau) = c(e - w_1, \theta)$.

   (a) If $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$, the politician's problem becomes:

$$\max_{\tau\in[0,+\infty)} V(\tau) = \begin{cases} \mu U_2 F(\Psi - \tau + \tau) - \nu\eta c^{-1}(1, D) & \\ \qquad -(1-\nu)\eta(\Psi - \tau - w_1) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [0, \tau_2] \\ \mu U_2 F(\Psi - \tau + \tau) - \nu\eta c^{-1}(1, D) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [\tau_2, \tau_3] \\ \mu U_2 F(\tau) - \tau - \nu\eta c^{-1}(1, D) + \mu\lambda - \Psi & \text{if } \tau \geq \tau_3 \end{cases}$$

Since $F(\Psi) = 1$, this can be written as:

$$\max_{\tau \in [0,+\infty)} V(\tau) = \begin{cases} \mu U_2 - \nu\eta c^{-1}(1,D) - (1-\nu)\eta(\Psi - w_1) & \\ \quad -(1-(1-\nu)\eta)\tau + \mu\lambda - \Psi & \text{if } \tau \in [0,\tau_2] \\ \mu U_2 - \nu\eta c^{-1}(1,D) - \tau + \mu\lambda - \Psi & \text{if } \tau \geq \tau_2 \end{cases}$$

Since $\nu > \bar{\nu}$, the first segment is decreasing in τ so it is maximized at $\tau = 0$. The second segment is decreasing in τ and therefore maximized at $\tau = \tau_2 = \Psi - w_1$. Therefore, it is optimal to set $\tau = 0$ when $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$ and $\mu\phi w_2 f(\Psi) \geq c(\Psi - \tau - w_1, \theta)$, so the politician prefers the informal policy ($V(0) > V(\Psi - w_1) > V(\Psi)$).

(b) If $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, \theta)$, the politician's problem becomes:

$$\max_{\tau \in [0,+\infty)} V(\tau) = \begin{cases} \mu U_2 \left[\nu F(\Psi - \tau + \tau) + (1-\nu) F\left(e_H^*(\tau) + \tau\right)\right] & \\ \quad -\nu\eta c^{-1}(1,D) - (1-\nu)\eta(e_H^*(\tau) - w_1) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [0,\tau_2] \\ \mu U_2 F(\Psi - \tau + \tau) - \nu\eta c^{-1}(1,D) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [\tau_2,\tau_3] \\ \mu U_2 F(\tau) - \tau - \nu\eta c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \geq \tau_3 \end{cases}$$

Since $F(\Psi) = 1$, this can be written as:

$$\max_{\tau \in [0,+\infty)} V(\tau) = \begin{cases} \mu U_2 \left[\nu + (1-\nu) F\left(e_H^*(\tau) + \tau\right)\right] - \nu\eta c^{-1}(1,D) & \\ \quad -(1-\nu)\eta(e_H^*(\tau) - w_1) - \tau + \mu\lambda - \Psi & \text{if } \tau \in [0,\tau_2] \\ \mu U_2 - \nu\eta c^{-1}(1,D) - \tau + \mu\lambda - \Psi & \text{if } \tau \geq \tau_2 \end{cases}$$

i. For $\tau \in [0, \tau_2]$, we know from Lemma 3 that when $\nu > \bar{\nu}$, the first segment is decreasing in τ. Therefore the first segment is maximized at $\tau = 0$ and its maximum is therefore:

$$\begin{aligned} V(0) = &\mu U_2 \left[\nu + (1-\nu) F\left(e_H^*(0)\right)\right] - \nu\eta c^{-1}(1,D) \\ &- (1-\nu)\eta(e_H^*(0) - w_1) + \mu\lambda - \Psi \end{aligned}$$

ii. When $\tau \geq \tau_2$, the derivative of the payoff function with respect to τ is $\frac{\partial V(\tau)}{\partial \tau} = -1$. This segment is therefore maximized at $\tau = \tau_2 = \Psi - w_1$ and since $V(0) > V(\tau_2)$, it is never optimal to set the tax in the interval $[\tau_2, +\infty)$.

Therefore when $\mu\phi w_2 f(\Psi) < c(\Psi - \tau - w_1, \theta)$, the informal policy is also preferred to the formal policy ($V(0) > V(\Psi - w_1) > V(\Psi)$).

Hence when $\phi < \frac{1}{\mu w_2 f(\Psi)}$, it is better for the politician to choose the informal policy with $\tau = 0$ if and only if inequality (5) is satisfied ($V(0) > V(\Psi)$). Instead, when $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$ it is always better for the politician to choose the informal policy with $\tau = 0$: $V(0) > V(\Psi - w_1) > V(\Psi)$.

Part 2: Next, we show the result in the Proposition: that there exists a threshold $\bar{\phi}_H$ such that the informal policy is chosen if and only if $\phi > \bar{\phi}_H$. We do this in three steps. Only the first step is necessary to show the existence of some threshold $\bar{\phi}_H$ such that an informal system is preferred if $\phi > \bar{\phi}_H$. Steps 2 and 3 are needed to show that this threshold is unique. The complication in showing uniqueness stems from the fact that, when the observability of public services (ϕ) increases, not only do the incentive of bureaucrats to fund services increase, which makes informal fiscal systems relatively more valuable, but the marginal value of increasing taxes to learn about the candidate's ability also increases (because taxes and ability are complement). This makes formal fiscal systems, with higher taxes, relatively more valuable. These two opposite effects imply that the value of informal systems relative to formal systems could be non-monotonic in ϕ. We show that, if the share of high-ability bureaucrats (μ) is high, the value of learning about the correct type is relatively lower, so the first effect dominates and the difference between the two systems strictly increases in ϕ.

1. **Step 1:** From Part 1, we can directly obtain that the politician prefers the informal policy at the highest value of $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$.
   Claim 1: At $\phi = 1$, $V(0) > V(\Psi)$.

   Proof. This follows directly from Part 1, Case 2: when $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$ and $\nu > \bar{\nu}$, the informal policy is strictly better than the formal policy, $V(0) > V(\Psi - w_1) > V(\Psi)$. □

2. **Step 2:** We rewrite inequality (5) as:

$$\Psi + \mu U_2 \left[\nu \left(F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) - 1 \right) + (1-\nu)\left(F(e_H^*(0)) - 1 \right) \right] - (1-\nu)\eta(e_H^*(0) - w_1) > 0 \quad (6)$$

   and show that, for $\frac{1}{\mu w_2 f(w_1)} \leq \phi < \frac{1}{\mu w_2 f(\Psi)}$, the left-hand side of inequality (6) is increasing in ϕ if μ is large enough and η is low enough. Let $LHS(\phi) = \Psi + \mu U_2 \left[\nu \left(F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) - 1 \right) + (1-\nu)\left(F(e_H^*(0)) - 1 \right) \right] - (1-\nu)\eta(e_H^*(0) - w_1)$.

Claim 2: $LHS(\phi)$ is increasing in ϕ for $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right)$.

Proof. The derivative of $LHS(\phi)$ is:

$$\begin{aligned}\frac{\partial LHS(\phi)}{\partial \phi} =& \mu\frac{\partial U_2}{\partial \phi}\left[\nu\left(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right)+(1-\nu)\left(F(e_H^*(0))-1\right)\right] \\ &+\mu U_2\left[\nu\frac{\partial F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}{\partial \phi}+(1-\nu)\frac{\partial F(e_H^*(0))}{\partial\phi}\right]-(1-\nu)\eta\frac{\partial e_H^*(0)}{\partial \phi}\end{aligned}$$

This is positive if:

$$\begin{aligned}&\nu\left[\mu U_2\frac{\partial F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}{\partial \phi}\right]+(1-\nu)\left[\mu U_2\frac{\partial F(e_H^*(0))}{\partial\phi}-\eta\frac{\partial e_H^*(0)}{\partial\phi}\right] \\ &> \mu\frac{\partial U_2}{\partial\phi}\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right)+(1-\nu)\left(1-F(e_H^*(0))\right)\right] \qquad (7)\end{aligned}$$

We first show that if η is small enough, then the left-hand side of inequality (7) is bounded below by a strictly positive number independent of μ and ϕ. For the first term of the left-hand side of inequality (7):

$$\begin{aligned}\mu U_2\frac{\partial F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}{\partial\phi} &= \mu U_2 f\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\left(\frac{1}{f'\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}\times-\frac{1}{\phi^2\mu w_2}\right) \\ &= \frac{\mu U_2}{-f'\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\phi^3\mu^2 w_2^2} > 0\end{aligned}$$

Given $U_2 = \lambda + \phi\lambda(1-\mu)$ and that f' is a continuous function on a compact set,

$$\lim_{\mu\to 1}\frac{\mu U_2}{-f'\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\phi^3\mu^2 w_2^2} > \frac{\lambda}{\|f'\|_\infty w_2^2} > 0$$

Next, we show that if η is small enough, the second term on the left-hand side of inequality (7) is strictly positive. The second term can be re-written as: $\mu U_2\frac{\partial F(e_H^*(0))}{\partial\phi} - \eta\frac{\partial e_H^*(0)}{\partial\phi} = \frac{\partial e_H^*(0)}{\partial\phi}\left(\mu U_2 f(e_H^*(0))-\eta\right)$. By applying implicit differentiation to $\mu\phi w_2 f(e) =$

$c(e - w_1, H)$, we obtain the derivative of e_H^* with respect to ϕ and can show that:

$$\frac{\partial e_H^*}{\partial \phi} = \frac{\mu w_2 f(e_H^*)^2}{c'(e_H^* - w_1, H) - \mu\phi w_2 f'(e_H^*)} > 0$$

Since $f'(\cdot) < 0$ by concavity of F and $c'(\cdot, H) > 0$ by convexity of C. Let

$$\bar{\eta} = \mu U_2 f(\Psi).$$

If $\eta < \bar{\eta}$, then $\mu U_2 f(e_H^*(0)) - \eta > \mu U_2 f(\Psi) - \eta > 0$ where the first inequality follows from the fact that $f(\Psi) < f(e)$ for any $e \in [0, \Psi)$ and the second directly from $\eta < \bar{\eta}$. Therefore if $\eta < \bar{\eta}$, then the second term of the left-hand side of inequality (7) is strictly positive. Note that the set of $\eta \in [1, \bar{\eta}]$ is non-empty since $\lambda f(\Psi) \geq \mu\lambda f(\Psi)$ and $\mu\lambda f(\Psi) > 1$ by assumption 2, so $\bar{\eta} > 1$.

Therefore, if $\eta < \bar{\eta}$, the left-hand side of inequality (7) is bounded below by a strictly positive number independent of μ and ϕ:

$$\lim_{\mu \to 1} \nu \left[\mu U_2 \frac{\partial F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}{\partial \phi} \right] + (1-\nu)\left[\mu U_2 \frac{\partial F(e_H^*(0))}{\partial \phi} - \eta \frac{\partial e_H^*(0)}{\partial \phi} \right] > \frac{\nu\lambda}{\|f'\|_\infty w_2^2} > 0$$

Finally, we show that the right-hand side of inequality (7) tends to 0 as μ tends to 1. Given $U_2 = \lambda + \phi\lambda(1-\mu)$, we have $\frac{\partial U_2}{\partial \phi} = \lambda(1-\mu)$. Therefore, as $\mu \to 1$, $\frac{\partial U_2}{\partial \phi} \to 0$. The other terms on the right-hand side of inequality (7), $f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$ and $e_H^*(0)$, remain bounded since they are continuous functions of μ on the compact set $[0, 1]$. Therefore, the right-hand side of inequality (7) tends to 0 as μ tends to 1.

We can therefore conclude that, if $\eta < \bar{\eta}$, there exists some μ close enough to 1 such that inequality 7 is satisfied. Let $\bar{\mu}_H$ the smallest value of μ such that inequality (7) is satisfied for any $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$ given some $\eta < \bar{\eta}$. □

3. **Step 3:** Finally, we show that, at the lowest value of $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$ the value of w_1 determines whether the politician prefers the formal of the informal policy.

Claim 3: At $\phi = \frac{1}{\mu w_2 f(w_1)}$, inequality (6) is satisfied if and only if $\mu U_2 F(w_1) > \mu U_2 - \Psi$.

Proof. At $\phi = \frac{1}{\mu w_2 f(w_1)}$, $e_H^* = e_D^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) = w_1$, so $LHS(\phi) = \Psi + \mu U_2(F(w_1) - 1)$. Therefore, $LHS(\phi) > 0 \Leftrightarrow \mu U_2 F(w_1) > \mu U_2 - \Psi$. □

Suppose that $\nu > \bar{\nu}$, $\eta < \bar{\eta}$ and $\mu > \bar{\mu}_H$, then combining claims 1, 2, and 3, we can conclude that:

1. If $\mu U_2 F(w_1) \geq \mu U_2 - \Psi$, $LHS(\phi) > 0$ for any $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, 1\right]$. So defining $\bar{\phi}_H = 0$, we have that the politician prefers an informal policy with high corruption if and only if $\phi \geq \bar{\phi}_H$.
2. If $\mu U_2 F(w_1) < \mu U_2 - \Psi$, then $LHS(\phi) < 0$ at $\phi = \frac{1}{\mu w_2 f(w_1)}$ (Claim 3), $LHS(\phi) > 0$ at $\phi = \frac{1}{\mu w_2 f(\Psi)}$, and $LHS(\phi)$ is increasing in ϕ for any $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$ (Claim 2). We can therefore apply the intermediate value theorem to conclude that there must exist some $\bar{\phi}_H \in \left(\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right)$ such that inequality (6) is satisfied if and only if $\phi > \bar{\phi}_H$. That is, the politician **chooses an informal policy** if and only if $\phi > \bar{\phi}_H$.

This proves the statement in Proposition 1. □

Proof of Proposition 2. Suppose that either (1) $\phi \leq \frac{1}{\mu w_2 f(w_1)}$ or that (2) $\phi > \frac{1}{\mu w_2 f(w_1)}$ but $\nu \leq \underline{\nu}$ for any $\phi \in \left[\frac{1}{f(w_1)\mu w_2}, \frac{1}{f(\Psi)\mu w_2}\right]$. As for the proof of Proposition 1, we first solve for the maximum of each segment and then show that there exists a unique threshold on ϕ such that the politician chooses an informal policy if ϕ is above this threshold.

<u>**Part 1:**</u> From Lemma 3, we know that when $\phi \leq \frac{1}{\mu w_2 f(w_1)}$ or when $\phi > \frac{1}{\mu w_2 f(w_1)}$ but $\nu \leq \underline{\nu}$, the politician prefers the informal policy with low corruption to the informal policy with high corruption. Since the politician's expected payoff is decreasing on $\tau \in [\tau_2, \tau_3]$, this segment is maximized at $\tau_2 = f^{-1}\left(\frac{1}{\phi \mu w_2}\right) - w_1$. However, if $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, then $\tau_2 \leq 0$. In this case, the segment is maximized at $\tau = 0$. To determine whether the formal policy is better than the informal policy, we therefore need to consider three cases.

1. **CASE 1:** $\phi \leq \frac{1}{\mu w_2 f(w_1)}$. In this case, the maximum of the informal policy is achieved at $\tau = 0$ since $\tau_2 \leq 0$:

$$V(0) = \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi \mu w_2}\right)\right) - \nu \eta c^{-1}(1, D) + \mu \lambda - \Psi$$

The maximum of the formal policy remains the same as in Proposition 1:

$$V(\Psi) = \mu U_2 - \Psi - \nu \eta c^{-1}(1, D) + \mu \lambda - \Psi$$

Therefore, the informal policy is preferred to the formal policy if:

$$\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) > \mu U_2 - \Psi \tag{8}$$

2. **CASE 2:** $\frac{1}{\mu w_2 f(w_1)} < \phi < \frac{1}{\mu w_2 f(\Psi)}$. In this case, the maximum of the informal policy is achieved at $\tau = \tau_2 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1$ since $\tau_2 > 0$:

$$V(\tau_2) = \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1, D) - \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right) + \mu\lambda - \Psi$$

The maximum of the formal policy remains:

$$V(\Psi) = \mu U_2 - \Psi - \nu\eta c^{-1}(1, D) + \mu\lambda - \Psi$$

Therefore, the informal policy is preferred to the formal policy if:

$$\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right) > \mu U_2 - \Psi \tag{9}$$

3. **CASE 3:** $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$. In this case, the maximum of the informal policy is achieved at $\tau = \tau_2 = \Psi - w_1$. We know from the proof of Proposition 1 (Part 1, Case 2) that the formal policy is never optimal. In this case, the informal policy with low corruption is optimal and the maximum expected payoff is:

$$V(\Psi - w_1) = \mu U_2 - \nu\eta c^{-1}(1, D) - (\Psi - w_1) + \mu\lambda - \Psi$$

Hence when $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, it is better for the politician to choose the informal policy with $\tau = 0$ if and only if inequality (8) is satisfied. When $\phi \in \left[\frac{1}{\mu w_2 f(\bar{w}_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$, it is better for the politician to choose the informal policy with $\tau = \tau_2$ if and only inequality (9) is satisfied ($V(0) > V(\Psi)$). Instead, when $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$ it is always better for the politician to choose the informal policy with $\tau = \tau_2 = \Psi - w_1$: $V(\Psi - w_1) > V(\Psi)$.

Part 2: Next, we show the result in the Proposition: that there exists a threshold $\bar{\phi}_L$ such that the informal policy is chosen if and only if $\phi > \bar{\phi}_L$. We do this in four steps.

1. **Step 1:** From Part 1, we can directly obtain that the politician prefers the informal policy at the highest value of $\phi \in [0, 1]$.

Claim 1: At $\phi = \frac{1}{\mu w_2 f(\Psi)}$, $V(\tau_2) > V(\Psi)$.

Proof. This follows directly from Part 1, Case 3: when $\phi > \frac{1}{\mu w_2 f(\Psi)}$, the informal policy is strictly better than the formal policy, $V(\tau_2) > V(\Psi)$. □

2. **Step 2:** We begin by rewriting inequality (8) as:

$$\mu U_2 \left(F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) - 1 \right) + \Psi > 0 \tag{10}$$

and show that, for $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, the left-hand side of inequality (8) is increasing in ϕ if μ is large enough. Let $LHS_{2A}(\phi) = \mu U_2 \left(F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) - 1 \right) + \Psi$.

Claim 2: $LHS_{2A}(\phi)$ is increasing in ϕ for $\phi \leq \frac{1}{\mu w_2 f(w_1)}$.

Proof. The derivative of $LHS_{2A}(\phi)$ is:

$$\frac{\partial LHS_{2A}(\phi)}{\partial \phi} = \mu \frac{\partial U_2}{\partial \phi} \left(F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) - 1 \right) + \mu U_2 \frac{\partial F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right)}{\partial \phi}$$

This is positive if:

$$\mu U_2 f\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) \frac{\partial f^{-1}\left(\frac{1}{\phi \mu w_2} \right)}{\partial \phi} > \mu \frac{\partial U_2}{\partial \phi} \left(1 - F\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) \right) \tag{11}$$

We first show that the left-hand side of inequality (11) is bounded below by a strictly positive number independent of μ and ϕ. Note that $\mu U_2 f\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) = \frac{\lambda + \phi \lambda (1-\mu)}{\phi w_2}$ and that $\frac{\partial f^{-1}\left(\frac{1}{\phi \mu w_2} \right)}{\partial \phi} = \frac{1}{-f'\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right)} \times \frac{1}{\phi^2 \mu w_2}$. Therefore,

$$\lim_{\mu \to 1} \mu U_2 f\left(f^{-1}\left(\frac{1}{\phi \mu w_2} \right) \right) \frac{\partial f^{-1}\left(\frac{1}{\phi \mu w_2} \right)}{\partial \phi} \geq \frac{\lambda}{w_2} \frac{1}{\|f'\|_\infty} \times \frac{1}{w_2} > 0$$

Finally, we note that, as in the proof of Proposition 2, the first term on the right-hand side of inequality (11) tends to 0 as μ tends to 1 while the other terms remain bounded. We can therefore conclude that there exists some μ close enough to 1 such that inequality (11) is satisfied. Let $\bar{\mu}_{L1}$ the smallest value of μ such that inequality (11) is satisfied for any $\phi \leq \frac{1}{\mu w_2 f(w_1)}$. □

3. **Step 3:** Similarly, we rewrite inequality (9) as:

$$\mu U_2\left(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right)+\Psi-f^{-1}\left(\frac{1}{\phi\mu w_2}\right)+w_1>0 \tag{12}$$

and show that, for $\frac{1}{\mu w_2 f(w_1)} \leq \phi < \frac{1}{\mu w_2 f(\Psi)}$, the left-hand side of inequality (12) is increasing in ϕ if μ is large enough. Let $LHS_{2B}(\phi) = \mu U_2\left(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right)+\Psi-f^{-1}\left(\frac{1}{\phi\mu w_2}\right)+w_1$.

Claim 3: $LHS_{2B}(\phi)$ is increasing in ϕ for $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right)$.

Proof. The derivative of $LHS(\phi)$ is:

$$\frac{\partial LHS(\phi)}{\partial\phi} = \mu\frac{\partial U_2}{\partial\phi}\left(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right)+\mu U_2\frac{\partial F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}{\partial\phi}-\frac{\partial f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}{\partial\phi}$$

This is positive if:

$$\frac{\partial f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}{\partial\phi}\left(\mu U_2 f\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right) > \mu\frac{\partial U_2}{\partial\phi}\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) \tag{13}$$

We first show that the left-hand side of inequality (13) is bounded below by a strictly positive number independent of μ and ϕ. As above, note that $\mu U_2 f\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) = \frac{\lambda+\phi\lambda(1-\mu)}{\phi w_2}$ and that $\frac{\partial f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}{\partial\phi} = \frac{1}{-f'\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)} \times \frac{1}{\phi^2\mu w_2}$. Therefore,

$$\begin{aligned}\lim_{\mu\to1}\frac{\partial f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}{\partial\phi}\left(\mu U_2 f\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-1\right) &\geq \frac{1}{\|f'\|_\infty}\times\frac{1}{\phi^2 w_2}\left(\frac{\lambda+\phi\lambda(1-\mu)-\phi w_2}{\phi w_2}\right)\\ &> \frac{1}{\|f'\|_\infty}\times\frac{1}{w_2^2}(\lambda-w_2)>0\end{aligned}$$

Finally, we note that, as in the proof of Proposition 2, the first term on the right-hand side of inequality (13) tends to 0 as μ tends to 1 while the other terms remain bounded. We can therefore conclude that there exists some μ close enough to 1 such that inequality (13) is satisfied. Let $\bar{\mu}_{L2}$ the smallest value of μ such that inequality (13) is satisfied for any $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right)$. □

4. **Step 4:** Finally, we show that, at the lowest value of $\phi \in [0,1]$ the formal policy is strictly better than the formal policy.

Claim 4: At $\phi = 0$, inequality (8) is satisfied.

Proof. At $\phi = 0$, $e_H^* = e_D^* = 0$ (since the marginal benefit of e is 0), so $LHS_{2A}(\phi) = \Psi - \mu U_2 < 0$ since $\mu U_2 - \Psi > 0$. □

Suppose that $\nu \leq \underline{\nu}$ and $\mu > \max\{\bar{\mu}_{L1}, \bar{\mu}_{L2}\} := \bar{\mu}_L$, then combining claims 1, 2, 3, and 4 we can conclude by applying the intermediate value theorem that:

1. If $\mu U_2 F(w_1) \geq \mu U_2 - \Psi$, there exists $\bar{\phi}_L \in \left[0, \frac{1}{\mu w_2 f(w_1)}\right]$ such that the politician prefers an informal policy with low corruption if and only if $\phi \geq \bar{\phi}_L$.
2. If $\mu U_2 F(w_1) < \mu U_2 - \Psi$, there exists $\bar{\phi}_L \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$ such that the politician prefers an informal policy with low corruption if and only if $\phi \geq \bar{\phi}_L$.

This proves the statement in Proposition 2. □

A.1.4 Selection

Proof of Proposition 3. From Lemma 1, we know that the politician retains the bureaucrat if and only if $s = 1$. The probability that a bureaucrat of type θ is retained is therefore $\mathbb{P}(s = 1 \mid e_\theta^*, b_\theta^*, \tau^*) = \phi F(e_\theta^* + \tau^*)$. When $\nu > \bar{\nu}$ and the observability of the public service is high enough, $\phi > \bar{\phi}_H$, we know from Proposition 1 that the politician chooses an informal policy with $\tau^* = 0$ and from Lemma 4 that the bureaucrat privately funds $e_D^*(0) = f^{-1}\left(\frac{1}{\mu\phi w_2}\right)$ if dishonest and $e_H^*(0)$ which solves $\phi\mu w_2 f(e_H^*(0)) = c(e_H^*(0) - w_1, H)$ if honest. The probability that a dishonest bureaucrat is retained is therefore: $\mathbb{P}(s = 1 \mid e_D^*, b_D^*, \tau^*) = \mu\phi F\left(e_D^*(0)\right)$ while the probability that an honest bureaucrat is retained is $\mathbb{P}(s = 1 \mid e_H^*, b_H^*, \tau^*) = \mu\phi F(e_H^*(0))$. We show that this probability is higher for a dishonest bureaucrat:

$$\begin{aligned}\mathbb{P}(s = 1 \mid e_D^*, b_D^*, \tau^*) \geq \mathbb{P}(s = 1 \mid e_H^*, b_H^*, \tau^*) \quad &\Leftrightarrow \quad \mu\phi F(e_D^*(0)) \geq \mu\phi F(e_H^*(0)) \\ &\Leftrightarrow \quad e_D^*(0) \geq e_H^*(0)\end{aligned}$$

Note that $e_D^*(0)$ solves $\phi\mu w_2 f(e_D^*(0)) = 1$ (provided that $e_D^*(0) < \Psi$) while $e_H^*(0)$ solves $\phi\mu w_2 f(e_H^*(0)) = c(e_H^*(0) - w_1, H)$ (provided that $e_H^*(0) < \Psi$). Therefore,

$$e_D^*(0) = f^{-1}\left(\frac{c(e_D^*(0) - w_1, D)}{\phi\mu w_2}\right) > f^{-1}\left(\frac{c(e_H^*(0) - w_1, H)}{\phi\mu w_2}\right) = e_H^*(0)$$

since f^{-1} is decreasing (by concavity of F) and $c(\cdot, D) < c(\cdot, H)$. Therefore, if $e^*_D(0), e^*_H(0) < \Psi$ or if $e^*_H(0) < \Psi$ and $e^*_D(0) = \Psi$, then $e^*_D(0) > e^*_H(0)$. If instead $e^*_H(0) = e^*_H(0) = \Psi$, then the two probabilities are equal. □

A.1.5 Welfare

We first characterize the equilibria for a politician facing no moral hazard or adverse selection. We define the cost of funding public services, denoted K, as the amount of funds taken from voters (either in the form of tax or bribes) and used towards funding public services (i.e., not kept by the bureaucrat).

Lemma 7. *A politician who can impose b and e and perfectly observe θ and ω chooses a formal policy with $\tau^*_{FB} = \Psi - w_1$ if $\omega = 1$, $\tau^*_{FB} = 0$ if $\omega = 0$, $b^*_{FB} = 0$ and $e^*_{FB} = w_1$. The expected amount of public services is $y_{FB} = \mu\Psi$ and the expected cost of funding public services is $K_{FB} = \mu(\Psi - w_1)$.*

Proof of Lemma 7.

Since the politician has perfect information, she selects a high-ability bureaucrat in the second period and sets the optimal tax level at $\tau = \Psi - w_2$ (since $\lambda f(\Psi) > \mu\lambda f \Psi) > 1$ by assumption 2). The first-period choice of tax therefore has no effect on the second period and we can ignore the second period when solving for the first-period choices. In addition, since the politician can perfectly contract the level of bribe and tax, the honesty of the bureaucrat is irrelevant for the politician's problem.

If the first-period bureaucrat is low-ability, the public service cannot be delivered and it is therefore optimal to set $\tau = b = 0$ and set any $e \in [0, w_1]$. If the first-period bureaucrat is high-ability, the politician solves:

$$\max_{e,\tau,b} \; V_{FB}(e, \tau, b) = \lambda F(\tau + e) - \tau - \eta b \quad \text{s.t.} \quad e \leq w_1 + b$$

First note that we cannot have $e < w_1$. If we did, then the politician could increase e at no cost to voters. For any given level of e such that $e \geq w_1$, it is then optimal to always set the budget constraint binding as otherwise the politician could decrease b further. Therefore, $e = b + w_1$ and the problem becomes:

$$\max_{\tau,b} \; V_{FB}(\tau, b) = \lambda F(\tau + b + w_1) - \tau - \eta b \quad \text{s.t. } b \geq 0$$

Since b and τ are substitute in the production of the public service, the politician chooses the funding method with the lowest marginal cost. Since $\eta > 1$, the marginal cost of funding the good through bribes is larger than the marginal cost of funding it through taxes, so the politician sets $b = 0$, $e = w_1$, and the optimal level of τ which solves:

$$\max_{\tau} V_{FB}(\tau, 0) = \lambda F(\tau + w_1) - \tau$$

This function is maximized at $\tau = \Psi - w_1$ since the derivative of the function above with respect to τ, $\lambda f(\tau+w_1)-1$, is greater than zero for all $\tau \in [0, \Psi - w_1]$. This follows from the fact that, for any $\tau \in [0, \Psi - w_1]$, $\lambda f(\tau+w_1)-1 > \mu\lambda f(\Psi - w_1 + w_1) - 1 = \mu\lambda f(\Psi) - 1 > 0$ where the last inequality follows from assumption 2. Therefore, the politician sets $\tau_{FB} = \Psi - w_1$, $b_{FB} = 0$, and $e_{FB} = w_1$. The amount of public services is $y = \Psi$ if $\omega = 1$ and $y = 0$ if $\omega = 0$, so the expected amount of public services is $y_{FB} = \mu\Psi$. The expected cost of funding public services is $K_{FB} = \mu(\Psi - w_1)$. □

Next, we compute the funding and bribe levels in the equilibrium with moral hazard and adverse selection characterized in Proposition 1.

Lemma 8. *When the politician chooses an informal policy with $\tau_P^* = 0$, the expected amount of bribes is $b_P^* = \nu c^{-1}(1, D) + (1-\nu)(e_H^*(0) - w_1)$ and the expected amount of public services is*

$$y_P = \begin{cases} \mu\Psi & \text{if } \phi\mu w_2 f(\Psi) \geq 1 \text{ and } \mu\phi w_2 f(\Psi) \geq c(\Psi - w_1, \theta) \\ \mu\left(\nu\Psi + (1-\nu)e_H^*(0)\right) & \text{if } \phi\mu w_2 f(\Psi) \geq 1 \text{ and } \mu\phi w_2 f(\Psi) < c(\Psi - w_1, \theta) \\ \mu\left(\nu f^{-1}\left(\frac{1}{\mu\phi w_2}\right) + (1-\nu)e_H^*(0)\right) & \text{if } \phi\mu w_2 f(\Psi) < 1 \end{cases}$$

and the expected social cost of funding public services is:

$$K_P = \begin{cases} \eta(\Psi - w_1) & \text{if } \phi\mu w_2 f(\Psi) \geq 1 \text{ and } \mu\phi w_2 f(\Psi) \geq c(\Psi - w_1, \theta) \\ \eta\left(\nu\Psi + (1-\nu)e_H^*(0) - w_1\right) & \text{if } \phi\mu w_2 f(\Psi) \geq 1 \text{ and } \mu\phi w_2 f(\Psi) < c(\Psi - w_1, \theta) \\ \eta\left(\nu f^{-1}\left(\frac{1}{\mu\phi w_2}\right) + (1-\nu)e_H^*(0) - w_1\right) & \text{if } \phi\mu w_2 f(\Psi) < 1 \end{cases}$$

When the politician chooses a formal policy with $\tau_P^* = \Psi$, the expected amount of bribes is $b_P^* = \nu c^{-1}(1, D)$, the expected amount of public services is $y_P = \mu\Psi$, and the expected social cost of funding public services is $K_P = \Psi$.

Proof of Lemma 8. The bribes and level of public services follow directly from Lemma 4 (when $\mu w_2 f(\Psi) < 1$) and Lemma 5 (when $\mu w_2 f(\Psi) \geq 1$) and the fact that $\tau_1 < 0 \leq \tau_2$ by assumption 1. The expected cost of funding public services is equal to the funding required for the amount of public services provided, minus the portion funded by the bureaucrats, multiplied by the marginal cost of the source of funding. Since the politician cannot observe the type of the bureaucrat the public services are funded whether or not they are delivered. The level of funding needed to fund an expected amount of services $\mu \times y$ is therefore y. The portion funded by the bureaucrats themselves depend on whether the private funding level is above or below their wage. If it is above, $e^*_\theta \geq w_1$, then the portion funded by bureaucrats is w_1. If it is below, $e^*_\theta < w_1$, the portion funded by the bureaucrat is the total amount of private funding, e^*_θ. The marginal cost of the source of funding is η if it comes from bribes and 1 if it comes from formal taxes. Since the funds only come from formal taxes in the formal policy (as $e^*_\theta = 0$) the portion funded by bureaucrats is 0 and the cost of funding $\mu\Psi$ is Ψ. In the informal policy, the funds are always above the bureaucrats' wage and come from bribes, which gives the result in the Lemma. □

Proof of Proposition 4. To prove the first part of the Proposition, note that with no moral hazard and adverse selection, the politician never chooses an informal policy (Lemma 7), whereas she does for some parameter values when facing agency distortions, i.e. moral hazard and adverse selection (Proposition 1). To prove the second part, we compare the first-best outcomes from Lemma 7 to the outcomes with a politician who faces moral hazard and adverse selection from Lemma 8.

1. When the politician chooses an informal policy, the expected amount of public services is either lower than the first best, since $\mu \left[\nu f^{-1}\left(\frac{1}{\phi\mu w_2}\right) + (1-\nu)e^*_H(0))\right] < \mu\left[\nu\Psi + (1-\nu)e^*_H(0))\right] < \mu\Psi = y_{FB}$, or it is the same as in the first best (when $y_P = y_{FB} = \mu\Psi$). When the amount is the same as in the first best, the cost of funding these services is $K_P = \eta(\Psi - w_1) > \Psi - w_1 > \mu(\Psi - w_1) = K_{FB}$. The expected amount of bribes in an informal policy is $b_P = \nu c^{-1}(1, D) + (1-\nu)(e^*_H(0) - w_1)$ (Lemma 8), while the amount of bribes in the first best is $b_{FB} = 0$ (Lemma 7). Therefore, in this case, agency distortions increase corruption and either strictly decrease the amount of public services ($y_P < \mu\Psi = y_{FB}$) or increase the cost of funding them ($K_P > K_{FB}$).
2. If she chooses a formal policy, the expected amount of public services is $y_P = \mu\Psi = y_{FB}$, the expected cost of funding public services is $K_P = \Psi - w_1 > \mu(\Psi - w_1)$, and the expected amount of bribes is $b_P = \nu c^{-1}(1, D) > 0 = b_{FB}$. Therefore in this case, agency distortions increase corruption and increase the cost of funding.

□

A.1.6 Political frictions

We first derive the equilibrium outcome when the politician maximizes the utility of group R using the results from Proposition 1. We define ν_R as the equivalent in this model of $\bar{\nu}$ in Lemma 3 and η_R, μ_R, and ϕ_R as the equivalents of $\bar{\eta}$, $\bar{\mu}_H$ and $\bar{\phi}_H$ in Proposition 1 (see the proof of Lemma 9 for the definition of these thresholds).

Lemma 9. *Suppose that $\nu > \nu_R$, $\eta < \eta_R$ and $\mu > \mu_R$. In equilibrium, a politician who favors group R implements an informal policy with $t^*_R = 0$ if ϕ is large enough ($\phi > \phi_R$). Otherwise, she implements a formal policy with $t^*_R = \frac{\Psi}{W_R+W_P}$. If the politician implements an informal policy, the expected amount of public services is $y_R = \mu\left[\nu f^{-1}\left(\frac{1}{\phi\mu w_2}\right) + (1-\nu)e^*_H(0))\right]$ and the expected amount of bribes is $b_R = \nu c^{-1}(1, D) + (1-\nu)(e^*_H(0) - w_1)$. If she implements a formal policy, the expected amount of public services is $y_R = \mu\Psi$ and the expected amount of bribes is $b_R = \nu c^{-1}(1, D)$.*

Proof of Lemma 9. Let $\tau = t(W_R + W_P)$ and $U^R_2 = \lambda_R + \phi\lambda_R(1-\mu)$. Using Lemma 4 to substitute the bureaucrat's optimal actions into the politician's objective function, we obtain the following politician problem:

$$\max_{\tau\in[0,+\infty)} V(\tau) = \begin{cases} \nu\left[\mu U^R_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \frac{\eta}{2}c^{-1}(1,D)\right] & \\ +(1-\nu)\left[\mu U^R_2 F(\tau + e^*_H(\tau)) - \frac{\eta}{2}(e^*_H(\tau) - w_1)\right] - \tau\frac{W_R}{W_R+W_P} + \mu\lambda - \Psi & \text{if } \tau\in[0,\tau_2] \\ \mu U^R_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\frac{\eta}{2}c^{-1}(1,D) - \tau\frac{W_R}{W_R+W_P} + \mu\lambda - \Psi & \text{if } \tau\in[\tau_2,\tau_3] \\ \mu U^R_2 F(\tau) - \tau\frac{W_R}{W_R+W_P} - \nu\frac{\eta}{2}c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \geq \tau_3 \end{cases}$$

The only differences in these expressions with those in the proof of Proposition 1 is that the cost of the tax is multiplied by $\frac{W_R}{W_R+W_P}$ to reflect the incidence on group R and the cost of corruption, η is divided by two. We show that the Proof of Lemma 3 and Proposition 1 can be applied by simply redefining the thresholds on parameters.

First note that, following the argument in the Proof of Lemma 3, the first segment is decreasing as long as $\nu > \nu_R$, where ν_R is defined as:

$$\nu_R = \max_{\tau\in[0,\tau_2]} \frac{\mu U^R_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \frac{\eta}{2}\frac{\partial e^*_H(\tau)}{\partial\tau} - \frac{W_R}{W_R+W_P}}{\mu U^R_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \frac{\eta}{2}\frac{\partial e^*_H(\tau)}{\partial\tau}}$$

Note that, unlike $\bar{\nu}$, it is possible for ν_R to be negative. This happens when $\mu U_2^R f(\tau + e_H^*(\tau))\left(1+\frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \frac{\eta}{2}\frac{\partial e_H^*(\tau)}{\partial \tau} - \frac{W_R}{W_R+W_P} \leq 0$ for all $\tau \in [0, \tau_2]$. In this case, the first segment is always decreasing. However, the denominator remains positive and larger than the numerator. The first segment is therefore decreasing in τ (and thus decreasing in t) if and only if $\nu \geq \nu_R$, where $\nu_R \in [0, 1)$. Since the second segment is also decreasing in τ (and thus in t), the maximum of the first two segments is obtained at $t = 0$. The maximum of the third segment is $\tau = \Psi$, which implies $t = \frac{\Psi}{W_R+W_P}$. To see this, note that the derivative of the third segment with respect to τ is $\mu f(\tau) U_2^R - \frac{W_R}{W_R+W_P}$. Given assumption 3, $\mu \lambda_R f(\Psi) - \frac{W_R}{W_R+W_P} > 0$, so $\mu U_2^R f(\Psi) - \frac{W_R}{W_R+W_P} > 0$ and therefore $\mu U_2^R f(\tau) - \frac{W_R}{W_R+W_P} > 0$ for any $\tau \leq \Psi$. The segment is therefore increasing up to the maximum level of tax $\tau = \Psi$.

Finally, following the proof of Proposition 1, if μ is large enough and η is small enough, the politician chooses an informal policy if ϕ is greater than some threshold $\bar{\phi}_R$. Specifically, the politician chooses an informal policy if:

$$\nu\left[\mu U_2^R F\left(f^{-1}\left(\frac{1}{\phi \mu w_2}\right)\right)\right] + (1-\nu)\left[\mu U_2^R F(e_H^*(0)) - \frac{\eta}{2}(e_H^*(0) - w_1)\right] > \mu U_2^R - \Psi \frac{W_R}{W_R + W_P} \tag{14}$$

The only differences with expression (5) in the Proof of Proposition 1 is that the last term is multiplied by $\frac{W_R}{W_R+W_P}$ and that η is replaced by $\frac{\eta}{2}$. Therefore, it is still the case that the difference between the two sides is increasing in ϕ for $\mu > \mu_R$ and $\eta < \eta_R := 2\mu U_2^R f(\Psi)$ as in the Proof of Proposition 1. It is also still the case that, at $\phi = \frac{1}{f(w_1)\mu w_2}$, the left-hand side is lower than the right-hand side if $\mu U_2^R F(w_1) < \mu U_2^R - \Psi \frac{W_R}{W_R+W_P}$. Finally, the left-hand side is greater than the right-hand side at $\phi = 1$ when $\nu > \nu_R$ (which is now equivalent to $\frac{2W_R}{W_R+W_P} > \eta(1-\nu)$ when $\phi \mu w_2 f(\Psi) \geq 1$). Therefore, we can conclude that, if $\mu U_2^R F(w_1) \geq \mu U_2^R - \Psi \frac{W_R}{W_R+W_P}$, the politician always prefers an informal policy, while if $\mu U_2^R F(w_1) < \mu U_2^R - \Psi \frac{W_R}{W_R+W_P}$, there exists $\bar{\phi}_R$ such that the politician chooses an informal system if and only if $\phi > \bar{\phi}_R$. □

We now solve the case of the social planner facing both moral hazard and adverse selection. Let ν_{SP}, η_{SP} and μ_{SP} denote three thresholds that are equivalent to the thresholds ν_R, η_R and μ_R in Lemma 9 but for the social planner.

Lemma 10. *Suppose that $\nu > \nu_{SP}$, $\eta < \eta_{SP}$ and $\mu > \mu_{SP}$. A social planner who maximizes the sum of the utilities of the two groups but cannot impose b and e implements an informal policy with $t_{SP}^* = 0$ if ϕ is large enough ($\phi > \phi_{SP}$). Otherwise, she implements a formal policy with $t_{SP}^* = \frac{\Psi}{W_R+W_P}$. When the social planner chooses an informal policy, the expected amount of public services is $y_{SP} = \mu\left[\nu f^{-1}\left(\frac{1}{\phi\mu w_2}\right) + (1-\nu)e_H^*(0)\right]$ and the expected amount of bribes is*

$b_{SP} = \nu c^{-1}(1, D) + (1 - \nu)(e^*_H(0) - w_1)$. *When she chooses a formal policy, the expected amount of public services is* $y_{SP} = \mu\Psi$ *and the expected amount of bribes is* $b_{SP} = \nu c^{-1}(1, D)$.

Proof of Lemma 10. Let $\tau = t(W_R + W_P)$ and $U_2^{SP} = (\lambda_R + \lambda_P) + \phi(\lambda_R + \lambda_P)(1 - \mu)$. Using Lemma 4, the social planner's problem becomes:

$$\max_{\tau \in [0,+\infty)} V_{SP}(\tau) = \begin{cases} \nu \left[\mu U_2^{SP} F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \eta c^{-1}(1, D)\right] \\ \quad +(1-\nu)\left[\mu U_2^{SP} F(\tau + e^*_H(\tau)) - \eta(e^*_H(\tau) - w_1)\right] \\ \qquad -\tau + \mu(\lambda_R + \lambda_P) - \Psi & \text{if } \tau \in [0, \tau_2] \\ \mu U_2^{SP} F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1, D) - \tau + \mu(\lambda_R + \lambda_P) - \Psi & \text{if } \tau \in [\tau_2, \tau_3] \\ \mu U_2^{SP} F(\tau) - \tau - \nu\eta c^{-1}(1, D) + \mu(\lambda_R + \lambda_P) - \Psi & \text{if } \tau \geq \tau_3 \end{cases}$$

The only differences in these expressions with those in the proof of Lemma 9 is that the cost of the tax is multiplied by 1, the cost of corruption is η and the benefit of public services is $\lambda_R + \lambda_P$. We can therefore follow the logic of the proof of Lemma 9 and apply Proposition 1 by defining:

$$\nu_{SP} = \max_{\tau \in [0,\tau_2]} \frac{\mu U_2^{SP} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2^{SP} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial \tau}}$$

and $\eta_{SP} = \mu U_2^{SP} f(\Psi)$ then we can conclude that there exists $\bar{\phi}_{SP}$ such that the politician chooses an informal system if and only if $\phi > \bar{\phi}_{SP}$. □

Proof of Proposition 5. To prove the Proposition, we compare the conditions for a social planner to choose an informal policy from Lemma 10 to the condition for a politician to choose an informal policy from Lemma 9.

First note that $\nu > \nu_{SP} \Rightarrow \nu > \nu_R$ since:

$$\begin{aligned} \nu_{SP} &= \max_{\tau \in [0,\tau_2]} \frac{\mu U_2^{SP} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial \tau} - 1}{\mu U_2^{SP} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \eta\frac{\partial e^*_H(\tau)}{\partial \tau}} \\ &\geq \max_{\tau \in [0,\tau_2]} \frac{\mu U_2^{R} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \frac{\eta}{2}\frac{\partial e^*_H(\tau)}{\partial \tau} - \frac{W_R}{W_R+W_P}}{\mu U_2^{R} f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right) - \frac{\eta}{2}\frac{\partial e^*_H(\tau)}{\partial \tau}} = \nu_R \end{aligned}$$

This follows from observing that, for any $\tau \in [0, \tau_2]$, (1) $f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial \tau}\right)$ (from

Lemma 6) so the two functions are increasing in U_2 and $U_2^{SP} > U_2^R$ and (2) $\frac{\partial e_H^*(\tau)}{\partial \tau} < 0$ so the two functions are increasing in η and $\eta > \frac{\eta}{2}$.

Second, note that $\eta < \eta_R \Rightarrow \eta < \eta_{SP}$ since: $\eta_R = 2\mu\lambda^R(1+\phi(1-\mu))f(\Psi) < \eta_{SP} = 2\mu(\lambda^R+\lambda^P)(1+\phi(1-\mu))f(\Psi)$. Therefore, when $\nu > \nu_{SP}$, $\eta < \eta_R$ and $\mu > \max\{\mu_R, \mu_{SP}\}$, the politician chooses an informal system when condition (14) in the proof of Lemma 9 is satisfied:

$$\nu\left[\mu U_2^R F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] + (1-\nu)\left[\mu U_2^R F(e_H^*(0)) - \frac{\eta}{2}(e_H^*(0)-w_1)\right] > \mu U_2^R - \Psi\frac{W_R}{W_R+W_P}$$

Instead, the social planner chooses an informal system when the following condition is satisfied:

$$\nu\left[\mu U_2^{SP} F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] + (1-\nu)\left[\mu U_2^{SP} F(e_H^*(0)) - \eta(e_H^*(0)-w_1)\right] > \mu U_2^{SP} - \Psi \tag{15}$$

Next, notice that if the social planner prefers the informal policy, then condition (15) implies that

$$\Psi - (1-\nu)\eta(e_H^*(0)-w_1) > \mu U_2^{SP}\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + (1-\nu)\left(1-F(e_H^*(0))\right)\right]$$

In addition, since $\lambda_P > \lambda_R$, then $U_2^{SP} = (\lambda_R+\lambda_P)(1+\phi(1-\mu)) > (\lambda_R+\lambda_R)(1+\phi(1-\mu)) = 2U_2^R$, so

$$\begin{aligned}\Psi - (1-\nu)\eta(e_H^*(0)-w_1) &> \mu U_2^{SP}\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + (1-\nu)\left(1-F(e_H^*(0))\right)\right]\\ &> 2\mu U_2^R\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + (1-\nu)\left(1-F(e_H^*(0))\right)\right]\end{aligned}$$

Therefore, we have:

$$\frac{1}{2}\left(\Psi - (1-\nu)\eta(e_H^*(0)-w_1)\right) > \mu U_2^R\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + (1-\nu)\left(1-F(e_H^*(0))\right)\right]$$

Finally, since $\frac{W_R}{W_R+W_P} > \frac{1}{2}$ (as $W_R > W_P$), then

$$\frac{W_R}{W_R+W_P}\Psi - (1-\nu)\frac{\eta}{2}(e_H^*(0)-w_1) > \frac{1}{2}\left(\Psi - (1-\nu)\eta(e_H^*(0)-w_1)\right)$$

Therefore,

$$\begin{aligned}\frac{W_R}{W_R+W_P}\Psi - (1-\nu)\frac{\eta}{2}(e_H^*(0)-w_1) &> \frac{1}{2}\left(\Psi - (1-\nu)\eta(e_H^*(0)-w_1)\right)\\ &> \mu U_2^R\left[\nu\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + (1-\nu)\left(1-F(e_H^*(0))\right)\right]\end{aligned}$$

Which is condition (14). Therefore, since $\bar{\phi}_R$ is the lowest value of ϕ such that inequality (14) is satisfied and $\bar{\phi}_{SP}$ is the lowest value of ϕ such that inequality (15), then $\phi > \bar{\phi}_{SP} \Rightarrow \phi > \bar{\phi}_R$, which implies that $\bar{\phi}_{SP} > \bar{\phi}_R$ which proves the statement. □

A.2 User fees

A.2.1 Setup

To understand the connection between user fees and informal fiscal systems, we consider the following model. Suppose that the bureaucrat can now set a minium fee, $\underline{\kappa}$, which the citizen needs to pay to access the public service. The citizens chooses a level of *user fee* $\kappa \in [0, \infty)$ to pay, knowing that the service will only be available if $\kappa \geq \underline{\kappa}$ but also that the bureaucrat can use the user fee κ to fund the service. We assume that bureaucrats cannot use bribes to fund public services but instead can use the user fee paid by the citizen κ. The timing of the the baseline model is modified as follows.

1. The politician chooses a level of tax τ anticipating how the user fees might increase the bureaucrat's funding but add an extra cost for the citizens. The second-period problem remains the same as in the baseline model, but the politician's first-period problem becomes:

$$\max_{\tau} \lambda F(\omega(\tau + e^*_\theta(\tau, \kappa^*(\underline{\kappa}))))\mathbb{1}(\kappa \geq \underline{\kappa}) - \tau - \kappa^*(\underline{\kappa}) - \eta b^*_\theta(\tau, \kappa^*(\underline{\kappa}))$$

2. The bureaucrat sets a minimum user fee, $\underline{\kappa}$, for the citizen to access the service, anticipating how this will affect the citizen's choice of user fee, $\kappa^*(\underline{\kappa})$, and the bureaucrat's own funding of the service $e^*(\tau, \kappa^*(\underline{\kappa}))$ and bribe-taking, $b^*(\tau, \kappa^*(\underline{\kappa}))$, in the final stage. The bureaucrat's problem at this stage is:

$$\max_{\underline{\kappa}} \quad w_1 + b + \kappa^*(\underline{\kappa}) - e^*(\tau, \kappa^*(\underline{\kappa})) + \mu\phi w_2 F(\tau + e^*(\tau, \kappa^*(\underline{\kappa}))) - C(b^*(\tau, \kappa^*(\underline{\kappa})), \theta)$$
$$\text{s.t.} \quad 0 \leq e \leq w_1 + \kappa^*(\underline{\kappa}), \; 0 \leq b$$

3. The citizen observes the level of tax τ and the minimum user fee $\underline{\kappa}$ and chooses how much user fee, κ, to pay towards the service anticipating how much the bureaucrat

will want to fund given the user fee and tax, $e^*_\theta(\tau,\kappa)$. The citizen's problem is:

$$\max_{\kappa} W(\kappa) = \lambda \mathbb{E}_{(\theta,\omega)}\left[\mathbb{1}\{\omega(\tau + e^*_\theta(\tau,\kappa)) \geq \bar{y}\}\mathbb{1}\{\kappa \geq \underline{\kappa}\} - \tau - \kappa - \eta b^*_\theta(\tau,\kappa)\right]$$

4. Finally, the bureaucrat chooses how much funding to provide for the public service, $e^*(\tau, \kappa^*(\underline{\kappa}))$, and bribes to take, $b^*(\tau, \kappa^*(\underline{\kappa}))$, given the tax level, τ, and the user fee paid by the citizen, κ. We assume that the politician receives a signal based on whether the bureaucrat met the threshold of public service provision, $\bar{y}$, rather than whether the public service was delivered, so the choice of $\underline{\kappa}$ does not affect the signal received by the politician. The user fee κ is therefore either used to relax the bureaucrat's budget constraint or to increase the bureaucrat's payoff, if the bureaucrat pockets part of the user fees.

A.2.2 Bureaucrat's choice of funding

By backward induction, we begin by solving the bureaucrat's choice of funding and bribe given some user fee κ and tax τ. The bureaucrat chooses e and b to solve:

$$\begin{aligned}\max_{b,e} \quad & w_1 + b + \kappa - e + \mu\phi w_2 F(\tau + e) - C(b,\theta) \\ \text{s.t.} \quad & 0 \leq e \leq w_1 + \kappa,\ 0 \leq b\end{aligned}$$

The choice of bribe is now independent of the choice of funding so can be solved independently. Taking the first-order condition and following similar arguments as in the proof of Lemma 2, the equilibrium bribes are $b^*_H(\tau,\kappa) = 0$ and $b^*_D(\tau,\kappa) = c^{-1}(1, D)$. Regarding the choice of funding, if the budget constraint is not binding, then the first-order condition on e gives: $\mu\phi w_2 f(\tau + e) = 1$. This implies that $e = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. If instead, the constraint binds, then the optimal funding can be backed out from the constraint: $e^* = w_1 + \kappa$. We therefore obtain the following result.

Result 1: the bureaucrat's choice of funding is independent of the bureaucrat's honesty, $e^*_H(\tau,\kappa) = e^*_D(\tau,\kappa) = e^*(\tau,\kappa)$, and satisfies:

$$e^*(\tau,\kappa) = \begin{cases} 0 & \text{if } \tau > f^{-1}\left(\frac{1}{\mu\phi w_2}\right) \\ f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau & \text{if } \tau \in \left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \kappa, f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right] \\ w_1 + \kappa & \text{if } \tau \leq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \kappa \end{cases}$$

A.2.3 Citizen's choice of user fee

The citizen chooses how much user fee to pay. The citizen takes into account two considerations. First, she anticipates that her user fee will influence the funding from the bureaucrat. Second, she only gets the service if her user fee is above the minimum set by the bureaucrat, $\underline{\kappa}$. We assume that the citizen is not forward looking and only chooses κ to maximize her first-period utility without taking into account its effect on the bureaucrat selection for the second period. The citizen maximizes her expected payoff, given the bureaucrat's funding as follows:

$$\max_{\kappa} W(\kappa) = \mathbb{1}\{\kappa \geq \underline{\kappa}\} \left(\mu\lambda F(\tau + e^*(\tau,\kappa))\right) - \kappa - \tau - \eta\mathbb{E}_\theta[b^*_\theta(\tau,\kappa)]$$

We can substitute $e^*(\tau,\kappa)$ from Result 1 in the objective function, for different values of κ:

$$W(\kappa) = \begin{cases} \mathbb{1}\{\kappa \geq \underline{\kappa}\}) \left(\mu\lambda F(\tau)\right) - \kappa - \tau - \eta\mathbb{E}_\theta[b^*(\tau,\kappa)] & \forall\kappa, \text{ if } \tau > f^{-1}\left(\frac{1}{\mu\phi w_2}\right) \\ \mathbb{1}\{\kappa \geq \underline{\kappa}\} \left(\mu\lambda F(\tau + w_1 + \kappa)\right) - \kappa - \tau - \eta\mathbb{E}_\theta[b^*(\tau,\kappa)] & \text{if } \kappa \leq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau \\ \mathbb{1}\{\kappa \geq \underline{\kappa}\} \left(\mu\lambda F(\tau + f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau)\right) - \kappa - \tau - \eta\mathbb{E}_\theta[b^*(\tau,\kappa)] & \text{if } \kappa \geq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau \end{cases}$$

Conditional on $\kappa \geq \underline{\kappa}$, the first segment is strictly decreasing in κ. The second segment can be either increasing or decreasing. If the maximizer is interior, it satisfies the following first-order condition: $\mu\lambda f(\tau+w_1+\kappa)-1 = 0$. The second-order condition is satisfied given that F is concave. This segment is therefore maximized at $\kappa = \kappa^M := f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau + w_1)$ if $\underline{\kappa} \leq f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau+w_1) \leq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau$. Finally, the third segment is also decreasing in κ and therefore maximized at $\kappa = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau$, provided that $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau \geq \underline{\kappa}$.

The objective function's maximizer is therefore determined as follows.

Case 1: If $\tau > f^{-1}\left(\frac{1}{\mu\phi w_2}\right)$, then $e^* = 0$, and the citizen pays the lowest possible user fees such that the service is provided if the payoff from getting the service is higher than the payoff from not getting it: $\mu\lambda F(\tau) - \underline{\kappa} - \tau - \eta\mathbb{E}_\theta[b^*(\tau,\kappa)] \geq 0 - \tau - \eta\mathbb{E}_\theta[b^*(\tau,\kappa)]$

$$\kappa^*(\underline{\kappa}) = \begin{cases} \underline{\kappa} & \text{if } \mu\lambda F(\tau) \geq \underline{\kappa} \\ 0 & \text{otherwise} \end{cases}$$

Case 2: If $\tau \in \left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1, f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right]$, then κ is always greater than $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau < 0$. This means that the budget constraint never binds and $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. The citizen therefore pays the lowest possible user fees such that the service is provided if the payoff from getting the service is higher than the payoff from not getting it:

$$\kappa^*(\underline{\kappa}) = \begin{cases} \underline{\kappa} & \text{if } \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) \geq \underline{\kappa} \\ 0 & \text{otherwise} \end{cases}$$

Case 3: If $\tau \leq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1$, then the bureaucrat's budget constraint binds without any user fees as $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau > w_1$. The citizen either pays the local maximizer $\kappa^M = f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau + w_1)$, the minimum user fee $\underline{\kappa}$ or no user fee, depending on the relative payoff of each option. We consider three cases.

1. If $\kappa^M \leq \underline{\kappa}$, then the unconstrained optimal user fee is less than the minimum user fee, so the citizen pays the minimum user fee provided that $\mu\lambda F(\tau + w_1 + \underline{\kappa}) - \underline{\kappa} - \tau - \eta\mathbb{E}_\theta[b^*(\tau, \underline{\kappa})] \geq 0 - \tau - \eta\mathbb{E}_\theta[b^*(\tau, 0)]$ and no user fee otherwise, so

$$\kappa^*(\underline{\kappa}) = \begin{cases} \underline{\kappa} & \text{if } \mu\lambda F(\tau + w_1 + \underline{\kappa}) \geq \underline{\kappa} \\ 0 & \text{otherwise} \end{cases}$$

2. If $\kappa^M \in \left(\underline{\kappa}, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau\right)$, then the citizen wants to pay more than the minimum user fee but not so much that the bureaucrat's budget constraint is relaxed, so $\kappa^*(\underline{\kappa}) = \kappa^M = f^{-1}\left(\frac{1}{\mu\lambda}\right) - \tau - w_1$. Since κ^M maximizes the objective function, conditional on getting access to the good, for any $\kappa \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau\right)$, then paying κ^M is better than paying no user fee and not getting access to the service.

3. If $\kappa^M \geq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau$, then the citizen wants to pay more than the minimum user fee and ensure that the bureaucrat can provide the maximum funding possible, so $\kappa^*(\underline{\kappa}) = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau$. This is also better than paying no user fees since the objective function, conditional on getting access to the good, is increasing for any $\kappa \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - \tau\right)$ in this case.

A.2.4 Bureaucrat's choice of minimum user fee

The bureaucrat's utility is strictly increasing in the user fee (since it contributes directly to his utility and helps relax his budget constraint). As a result, the bureaucrat always sets the minimum user fee at the maximum level that the user would be willing to pay.

Case 1: If $\tau > f^{-1}\left(\frac{1}{\mu\phi w_2}\right)$, the bureaucrat is better-off when the citizen pays the fee and accesses the service so sets the minimum fee at the highest level that the citizen is willing to pay, i.e., $\underline{\kappa} = \mu\lambda F(\tau)$.

Case 2: If $\tau \in \left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1, f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right]$, the bureaucrat also sets the minimum fee at the highest level that the citizen is willing to pay, i.e., $\underline{\kappa} = \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right)$.

Case 3: If $\tau \leq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1$, there are two possibilities:

1. If $\lambda < \phi w_2$, then $f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau + w_1) < f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - (\tau + w_1)$. This means that the citizen values the provision of the good less than the bureaucrat does, and, as a result, the citizen would not be willing to fully alleviate the bureaucrat's constraint (that is, $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau \geq w_1 + \kappa^*(\underline{\kappa})$), so the constraint is always binding and $e^* = w_1 + \kappa^*(\underline{\kappa})$. In this case, $\kappa^*(\underline{\kappa})$ satisfies:

$$\kappa^*(\underline{\kappa}) = \begin{cases} \kappa^M = f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau + w_1) & \text{if } \underline{\kappa} < \kappa^M \\ \underline{\kappa} & \text{if } \kappa^M \leq \underline{\kappa} \leq \mu\lambda F(\tau + w_1 + \underline{\kappa}) \\ 0 & \text{if } \underline{\kappa} > \mu\lambda F(\tau + w_1 + \underline{\kappa}) \end{cases}$$

   Note that κ^M is strictly less than the value of $\underline{\kappa}$ such that $\underline{\kappa} = \mu\lambda F(\tau + w_1 + \underline{\kappa})$. This is because κ^M maximizes $\mu\lambda F(\tau + w_1 + \kappa) - \kappa$, and this maximum is strictly positive (otherwise $\kappa = 0$ would be optimal). As a result, $\mu\lambda F(\tau + w_1 + \kappa^M) - \kappa^M > 0 = \mu\lambda F(\tau + w_1 + \underline{\kappa})$ for $\underline{\kappa} = \mu\lambda F(\tau + w_1 + \underline{\kappa})$. Finally, since $\mu\lambda F(\tau + w_1 + \kappa) - \kappa$ is single-peaked, it is decreasing for $\kappa > \kappa^M$, so κ^M must be strictly less than the value of $\underline{\kappa}$ such that $\underline{\kappa} = \mu\lambda F(\tau + w_1 + \underline{\kappa})$. As a result, the optimal minimum user fee in this case is $\underline{\kappa}$ such that $\underline{\kappa} = \mu\lambda F(\tau + w_1 + \underline{\kappa})$ (this value exists since $F(\cdot)$ is strictly concave).

2. If $\lambda \geq \phi w_2$, then $f^{-1}\left(\frac{1}{\mu\lambda}\right) - (\tau + w_1) \geq f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - (\tau + w_1)$. This means that the citizen values the provision of the good more than the bureaucrat does, and, as a result, the citizen is willing to fully alleviate the bureaucrat's constraint (that is, $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau \leq w_1 + \kappa^*(\underline{\kappa})$), so the constraint is not binding and $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. In this case, $\kappa^*(\underline{\kappa})$ satisfies:

$$\kappa^*(\underline{\kappa}) = \begin{cases} f^{-1}\left(\frac{1}{\mu\lambda}\right) & \text{if } \underline{\kappa} < f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1 \\ \underline{\kappa} & \text{if } f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1 \leq \underline{\kappa} \leq \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) \\ 0 & \text{if } \underline{\kappa} > \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) \end{cases}$$

Note that $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1 < \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right)$ (given that the function $\lambda\mu F(\kappa + \tau + w_1)$ is maximized and positive at κ^M, it is positive for any $\kappa < \kappa^M$, and in particular at $\kappa = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1 < \kappa^M$). Therefore, the optimal $\underline{\kappa}$ in this case is $\underline{\kappa} = \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right)$.

Result 2: the value $\underline{\kappa}$ that can be extracted from the citizen is proportional to the citizen's value from the public good, λ, and is defined as follows

$$\underline{\kappa} = \begin{cases} \mu\lambda F(\tau) & \text{if } \tau > f^{-1}\left(\frac{1}{\mu\phi w_2}\right) \\ \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) & \text{if } \tau \in \left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1, f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right] \\ \underline{\kappa} \text{ s.t. } \underline{\kappa} = \mu\lambda F(\underline{\kappa} + w_1 + \tau) & \text{if } \tau \le f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 \text{ and } \lambda < \phi w_2 \\ \mu\lambda F\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) & \text{if } \tau \le f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 \text{ and } \lambda \ge \phi w_2 \end{cases}$$

In particular, when $\lambda < \phi w_2$, the bureaucrat would want to provide a higher level of funding but is constrained by the amount of user fee that the citizen is willing to pay.

A.2.5 Politician's choice of fiscal system

To reduce the number of cases, we make two assumptions. The first assumption is similar to assumption 2 in the baseline model:

Assumption 4. *The marginal benefit of increasing tax at the maximum tax level* Ψ *satisfies* $\mu(U_2 - \lambda)f(\Psi) - 1 > 0$.

The second allows us to focus on informal policies with high corruption when the citizen values the service relatively less than the bureaucrat:

Assumption 5. *The derivative of the first segment of the politician's payoff function when* $\lambda < \phi w_2$ *is negative for any* $\tau \in \left[0, f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right]$*:* $\mu(U_2 - \lambda)f(w_1 + \tau + \underline{\kappa}(\tau))\left(\frac{\partial\underline{\kappa}(\tau)}{\partial\tau} + 1\right) - 1 < 0$.

The politician solves:

$$\max_{\tau\in[0,+\infty)} V(\tau) = \begin{cases} \mu U_2 F\left(w_1+\tau+\underline{\kappa}(\tau)\right) - \nu\eta c^{-1}(1,D) & \\ \quad -\mu\lambda F\left(w_1+\tau+\underline{\kappa}(\tau)\right) - \tau + \mu\lambda - \Psi & \text{if } \tau \le f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 \text{ and } \lambda < \phi w_2 \\ \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1,D) & \\ \quad -\mu\lambda F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \tau + \mu\lambda - \Psi & \text{if } \tau \le f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 \text{ and } \lambda \ge \phi w_2 \\ \mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \nu\eta c^{-1}(1,D) & \\ \quad -\mu\lambda F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) - \tau + \mu\lambda - \Psi & \text{if } \tau \in \left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1, f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right] \\ \mu U_2 F(\tau) - \mu\lambda F(\tau) - \tau - \nu\eta c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right) \\ \mu U_2 - \mu\lambda - \tau - \nu\eta c^{-1}(1,D) + \mu\lambda - \Psi & \text{if } \tau \ge \Psi \end{cases}$$

Where $\underline{\kappa}(\tau)$ in the first segment solves $\underline{\kappa}(\tau) = F\left(w_1+\tau+\underline{\kappa}(\tau)\right)$.

The optimal tax within each segment is then:

1. **First segment:** if $\tau \le f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1$, then
   - If $\lambda < \phi w_2$, $\frac{\partial V(\tau)}{\partial\tau} = \mu(U_2-\lambda)f(w_1+\tau+\underline{\kappa}(\tau))\left(\frac{\partial\underline{\kappa}(\tau)}{\partial\tau}+1\right) - 1$. Given assumption 5, this segment is decreasing so it is maximized at $\tau^* = 0$.
   - If $\lambda \ge \phi w_2$, $\frac{\partial V(\tau)}{\partial\tau} = -1$, so this segment is decreasing and maximized at $\tau^* = 0$.
2. **Second segment:** if $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1, f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)$, the derivative of the function is $\frac{\partial V(\tau)}{\partial\tau} = -1$, so the segment is maximized at $\tau^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1$.
3. **Third segment:** if $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right)$, the derivative of the function is $\frac{\partial V(\tau)}{\partial\tau} = \mu(U_2-\lambda)f(\tau) - 1$. Given assumption 4, this is positive for any $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right)$, so $\tau^* = \Psi$.
4. **Third segment:** if $\tau \ge \Psi$, then the objective function is decreasing in τ for any τ, so $\tau^* = \Psi$.

As a result, the optimal tax is either $\tau^* = 0$ or $\tau^* = \Psi$.

- If $\lambda < \phi w_2$, the politician chooses $\tau^* = 0$ if and only if:

$$\begin{aligned}
&\mu(U_2-\lambda)F\left(w_1+\underline{\kappa}(0)\right)-\nu\eta c^{-1}(1,D)+\mu\lambda-\Psi \\
&\qquad > \mu(U_2-\lambda)\lambda-\Psi-\nu\eta c^{-1}(1,D)+\mu\lambda-\Psi \\
\Leftrightarrow\quad &\mu(U_2-\lambda)F\left(w_1+\underline{\kappa}(0)\right) > \mu(U_2-\lambda)-\Psi \\
\Leftrightarrow\quad &\Psi > \mu(U_2-\lambda)\left(1-F\left(w_1+\underline{\kappa}(0)\right)\right)
\end{aligned}$$

- If $\lambda \geq \phi w_2$, The politician chooses $\tau^* = 0$ if and only if:

$$\begin{aligned}
&\mu(U_2-\lambda)F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)-\nu\eta c^{-1}(1,D)+\mu\lambda-\Psi \\
&\qquad > \mu(U_2-\lambda)\lambda-\tau-\nu\eta c^{-1}(1,D)+\mu\lambda-\Psi \\
\Leftrightarrow\quad &\mu(U_2-\lambda)F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) > \mu(U_2-\lambda)-\Psi \\
\Leftrightarrow\quad &\Psi > \mu(U_2-\lambda)\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right)
\end{aligned}$$

  and chooses $\tau^* = \Psi$ otherwise.

When $\tau^* = 0$, the bureaucrat redistributes all of the user fee towards the public service. When $\tau^* = \Psi$, the bureaucrat keeps all the user fee to himself. When $\lambda \geq \phi w_2$, the choice of a user fee system vs. a formal system depends on the value that the citizen assigns to the public service, λ, rather than the observability of the service, ϕ (as in informal system).

Given these equilibria, the two results below follow directly from examining the conditions for the optimal tax and the resulting equilibrium funding, e^*:

Result 3: The optimal tax is independent of the share of dishonest bureaucrats ν.

Result 4: The probability of being re-selected is the same for any $\theta \in \{H, D\}$.

A.2.6 Redistribution with user fees.

Suppose that there are two groups, the rich, R, and the poor, P, as in Section 5.5. Assume that $\lambda_P > \phi w_2$, while $\lambda_R = 0 < \phi w_2$. Using the results above, we find that:

- For group P, if $\Psi > \mu(U_2-\lambda_P)\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) = \mu(1-\mu)\phi\lambda_P\left(1-F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right)$, then the politician chooses a system with user fees only, so $\tau^* = 0$ and the citizens from that group are happy to pay the minimum user fee which is equal to $\underline{\kappa} = \mu\lambda_P F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)$.

- For group R, assumption 5 no longer holds. Since $\lambda_R = 0$, the derivative of the third segment is decreasing everywhere, so the payoff function is decreasing for any τ. The politician therefore chooses a system with user fees only, so $\tau^* = 0$, the bureaucrat cannot extract any user fee so sets $\underline{\kappa} = 0$, and the citizens from that group prefer not to access the service, so $\kappa^*(\underline{\kappa}) = 0$.

As a result, there is no redistribution between the groups: the rich do not pay any taxes or user fees, while the poor pay a user fee that funds the service. With an informal fiscal system, instead, the rich would have paid a bribe, $b^*_\theta > 0$, and this bribe would have been partially used to fund the service.

A.3 Informal taxation

To understand the connection between informal taxation and informal fiscal systems, we consider the following model. Suppose that the citizen can contribute some amount κ to the public service funding after observing the tax (τ), bureaucrat funding (e), and the need ($\bar{y}$). We interpret this amount κ as informal taxation: funds or contributions in kind provided by the population separately from the government funding. The assumption that they can observe the need before choosing their funding captures the fact that informal taxation takes advantage of local information available to citizens but not necessarily to the state. However, we assume that this funding comes at a marginal cost of $\rho > 1$. This captures the fact that, unlike government and bureaucrat funding, the citizen's funding cannot take advantage of economies of scale or implementation experience. We solve this model by backward induction, starting from the citizen's choice of funding.

Citizen choice of funding. The citizen observes the tax (τ), the bureaucrat funding (e), the bureaucrat's ability (ω), and the need ($\bar{y}$) and chooses κ to maximize:

$$W(\kappa) = \lambda\mathbb{1}\{\omega(e+\tau) + \kappa \geq \bar{y}\} - \tau - \rho\kappa - \mathbb{E}_\theta[b_\theta]$$

If $\omega(e+\tau) \geq \bar{y}$, then this function is decreasing in κ, so it is optimal to set $\kappa^* = 0$. If $\omega(e+\tau) < \bar{y}$, then the function is decreasing in κ but jumps up discontinuously at

$\kappa = \bar{y} - \omega(e+\tau)$. The maximum is therefore either attained at $\kappa = 0$ or $\kappa = \bar{y} - \omega(e+\tau)$. In particular, the citizen chooses $\kappa = \bar{y} - \omega(e+\tau)$ if

$$\lambda - \tau - \rho(\bar{y} - \omega(e+\tau)) - \mathbb{E}_\theta[b_\theta] \geq -\tau - \mathbb{E}_\theta[b_\theta] \Leftrightarrow \lambda \geq \rho(\bar{y} - \omega(e+\tau))$$

and chooses $\kappa = 0$ otherwise. To reduce the number of cases to consider, we assume that $\lambda \geq \rho\bar{y}$, so this condition is always satisfied. This implies that the citizen's funding is $\kappa^*(\tau, e) = \max\{\bar{y} - (e+\tau), 0\}$.

Bureaucrat choice of funding and bribes. Note that, since only the tax and bureaucrat funding are informative about the bureaucrat's ability, the politician's signal is not affected by the citizen's funding. As a result, the signal structure and the bureaucrat's incentives are the same as in the baseline model, and the bureaucrat's choices are the same as in Lemma 2. To reduce the number of cases, we assume that $f^{-1}\left(\frac{1}{\phi\mu w_2}\right) < \min\{w_1, \Psi\}$, so that only Lemma 4 applies and only informal fiscal systems with low corruption are possible. As a result, the bureaucrat's funding and bribes are: $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau,\ \forall\theta \in \{H, D\}$, $b^*_D(\tau) = c^{-1}(1, D)$, and $b^*_H(\tau) = 0$ if $\tau < f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, and $e^*_\theta(\tau) = 0,\ \forall\theta \in \{H, D\},\ b^*_D(\tau) = c^{-1}(1, D),\ b^*_H(\tau) = 0$ if $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$.

Politician choice of tax. The politician's re-selection rule is the same as in Lemma 1 and the second-period choice of tax is the same as derived in the proof of Proposition 1. Anticipating the bureaucrat's funding $e^*_\theta(\tau)$ and the citizen's funding $\kappa^*(\tau, e)$, the politician's problem becomes:

$$\begin{aligned}\max_\tau V(\tau) = \mu\Bigg(&F(\tau+e)\left[\lambda + (\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D)\right] \\ &+ (1 - F(\tau+e))\left[\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) - \int_{\bar{y}=\tau+e}^{\Psi} \frac{\rho(\bar{y} - \tau - e) f(\bar{y})}{1 - F(\tau+e)} d\bar{y}\right]\Bigg) \\ &+ (1-\mu)\left(\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) - \int_{\bar{y}=0}^{\Psi} \rho\bar{y} f(\bar{y}) d\bar{y}\right)\end{aligned}$$

Case 1: When $\tau < f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ so the problem becomes:

$$\begin{aligned}\max_\tau V(\tau) =& \mu\Bigg(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\left[\lambda + (\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D)\right] \\ &+ \left(1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right)\Bigg[\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) \\ &- \int_{\bar{y}=f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}^{\Psi} \rho\left(\bar{y} - f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\frac{f(\bar{y})}{1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)}d\bar{y}\Bigg]\Bigg) \\ &+ (1-\mu)\left(\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) - \int_{\bar{y}=0}^{\Psi}\rho\bar{y}f(\bar{y})d\bar{y}\right)\end{aligned}$$

Taking the derivative with respect to τ gives $\frac{\partial V(\tau)}{\partial\tau} = -1 < 0$, so it is optimal to set $\tau^* = 0$ on this segment.

Case 2: When $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, $e^*_\theta(\tau) = 0$, so the maximization problem becomes:

$$\begin{aligned}\max_\tau V(\tau) =& \mu\Bigg(F(\tau)\left[\lambda + (\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D)\right] \\ &+ (1 - F(\tau))\left[\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) - \int_{\bar{y}=\tau}^{\Psi}\rho(\bar{y} - \tau)\frac{f(\bar{y})}{1 - F(\tau)}d\bar{y}\right]\Bigg) \\ &+ (1-\mu)\left(\lambda + (\mu\lambda - \Psi) - \tau - \eta\nu c^{-1}(1, D) - \int_{\bar{y}=0}^{\Psi}\rho\bar{y}f(\bar{y})d\bar{y}\right)\end{aligned}$$

Simplifying gives:

$$\begin{aligned}\max_\tau V(\tau) =& \lambda - \tau - \eta\nu c^{-1}(1, D) + \mu(1-\mu)\lambda F(\tau) + \mu\lambda - \Psi \\ &- \int_{\bar{y}=\tau}^{\Psi}\rho(\bar{y} - \tau)f(\bar{y})d\bar{y} - (1-\mu)\int_{\bar{y}=0}^{\Psi}\rho\bar{y}f(\bar{y})d\bar{y}\end{aligned}$$

Taking the derivative with respect to τ using the Leibnitz rule, gives:

$$\begin{aligned}\frac{\partial V(\tau)}{\partial\tau} &= -1 + \mu(1-\mu)\lambda f(\tau) - \rho\left[-(\tau - \tau)f(\bar{y})\frac{\partial\tau}{\partial\tau} + \int_{\bar{y}=\tau}^{\Psi} -1 \times f(\bar{y})d\bar{y}\right] \\ &= \mu(1-\mu)\lambda f(\tau) - 1 + \rho\left[F(\Psi) - F(\tau)\right]\end{aligned}$$

This derivative is strictly decreasing in τ (since $F(\cdot)$ is increasing and concave) but can be positive or negative. There are three possible cases:

- If $\frac{\mu(1-\mu)\lambda}{\phi\mu w_2} - 1 + \rho\left[1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] < 0$, then $\frac{\partial V(\tau)}{\partial \tau} < 0$ for any $\tau \in \left[f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right]$, so the second segment is maximized at $\tau^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$.
- If $\frac{\mu(1-\mu)\lambda}{\phi\mu w_2} - 1 + \rho\left[1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] > 0$ and $\mu(1-\mu)\lambda f(\Psi) - 1 < 0$ then by the intermediate value theorem, there exists $\tau^* \in \left[f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right]$ such that $\frac{\partial V(\tau)}{\partial \tau}\Big|_{\tau^*} = 0$ and the segment is maximized at τ^*.
- If $\frac{\mu(1-\mu)\lambda}{\phi\mu w_2} - 1 + \rho\left[1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right] > 0$ and $\mu(1-\mu)\lambda f(\Psi) - 1 > 0$, then $\frac{\partial V(\tau)}{\partial \tau} > 0$ for any $\tau \in \left[f^{-1}\left(\frac{1}{\phi\mu w_2}\right), \Psi\right]$, so the second segment is maximized at $\tau^* = \Psi$.

To simplify the analysis, we focus on the case where $\mu(1-\mu)\lambda f(\Psi) - 1 > 0$, so that the second segment is maximized at $\tau^* = \Psi$. In this case, the politician would choose $\tau^* = 0$ over $\tau^* = \Psi$ if and only if:

$$\begin{aligned} V(0) =& \lambda - \eta\nu c^{-1}(1, D) + \mu\lambda - \Psi + \mu\Bigg(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\lambda(1-\mu) \\ &- \int_{\bar{y}=f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}^{\Psi} \rho\left(\bar{y} - f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) f(\bar{y})d\bar{y}\Bigg) - (1-\mu)\left(\int_{\bar{y}=0}^{\Psi} \rho\bar{y} f(\bar{y})d\bar{y}\right) \\ &\geq V(\Psi) = \lambda - \Psi - \eta\nu c^{-1}(1, D) + \mu\lambda - \Psi + \mu(1-\mu)\lambda - (1-\mu)\left(\int_{\bar{y}=0}^{\Psi} \rho\bar{y} f(\bar{y})d\bar{y}\right) \end{aligned}$$

Or equivalently,

$$\begin{aligned} &\mu\Bigg(F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\lambda(1-\mu) - \int_{\bar{y}=f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}^{\Psi} \rho\left(\bar{y} - f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) f(\bar{y})d\bar{y}\Bigg) \geq -\Psi + \mu(1-\mu)\lambda \\ &\Leftrightarrow \Psi \geq \mu(1-\mu)\lambda\left(1 - F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right) + \int_{\bar{y}=f^{-1}\left(\frac{1}{\phi\mu w_2}\right)}^{\Psi} \rho\left(\bar{y} - f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right) f(\bar{y})d\bar{y}\Bigg) \end{aligned} \tag{16}$$

If this condition is satisfied, then the politician sets $\tau^* = 0$ and we have a system in which (1) Bureaucrats fund services privately, since $e^* = f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, (2) informal taxation occurs with some probability (i.e. when $\bar{y} > e^*$ or when $\omega = 0$), (3) there is no formal taxation. If the condition is not satisfied, then we have a system in which bureaucrats provide no funding, $e^* = 0$ and informal taxation only occurs when the bureaucrat is incompetent ($\omega = 0$).

Finally, note that a higher ρ increases the right-hand side of inequality (16) and therefore makes a formal system with relatively little informal taxation more likely. This is the case when the government's expertise or the bureaucrat's know-how are very valuable which makes informal taxation relatively less attractive.

A.4 No heterogeneity in ability

Suppose that there is no heterogeneity in ability: both bureaucrats are high ability and able to deliver the public service so $\omega = 1$.[33] We show that, in this case, there is a range of equilibria in which neither type of bureaucrat funds public services. In addition, the only equilibrium in which bureaucrats fund services does not survive a small perturbation of the politician's preference over the bureaucrat who is re-selected.

Proposition 6. *Suppose that $\omega = 1$ for all bureaucrats. Then there exists a set of equilibria in which $e^*_\theta = 0$.*

Proof of Proposition 6. If all bureaucrats have the same ability, the politician is indifferent between re-selecting the existing bureaucrat or firing the bureaucrat, independent of the bureaucrat's performance, y. Indeed, the politician's expected utility from re-selecting the bureaucrat in the second period is $\tilde{V}(r) = \max_\tau \{\lambda F(\tau) - \tau\}$, independent of the retention rule used. Next, we show that, in this context, both types of bureaucrats funding no public services is part of an equilibrium for any re-selection rule of the politician. Since the politician is indifferent between re-selecting the bureaucrat or not for any signal s, the following cases are possible:[34]

1. Suppose that the politician either always re-selects the bureaucrat for any s or never re-selects the bureaucrat for any s. In these cases, the bureaucrat maximizes the following expected utility by choosing funding e and bribes b in the first period: $u(e, b \mid \theta) = w_1 + b - e + w_2 - C(b, \theta)$, subject to $0 \leq e \leq w_1 + b$ and $0 \leq b$. Compared to the baseline model, the term $\mu\phi w_2 F(\tau + e)$ becomes w_2 since the bureaucrat is always re-selected for the second period. The solution to this problem is $e_\theta(\tau) = 0$, $b_H(\tau) = 0$ and $b_D(\tau) = c^{-1}(1, \theta)$ for any $\tau \in [0, 1 + \infty)$.

2. Suppose that the politician re-selects the bureaucrat if and only if $s = 0$. In this case, the bureaucrat maximizes the following expected utility: $u(e, b \mid \theta) = w_1 + w_2(1 -$

[33] If all bureaucrats were low ability, $\omega = 0$, then the public service would never be delivered so neither the bureaucrats nor the government would ever want to fund public services.

[34] We focus on pure strategy equilibria for simplicity, but the logic would extend to cases where the politician mixes between re-selecting and not re-selecting.

$\phi F(\tau + e)) + b - e - C(b, \theta)$, subject to $0 \leq e \leq w_1 + b$ and $0 \leq b$. Since this function is everywhere decreasing in e, the solution to this problem is also $e_\theta(\tau) = 0$, $b_H(\tau) = 0$ and $b_D(\tau) = c^{-1}(1, \theta)$ for any $\tau \in [0, 1 + \infty)$.

3. Finally, suppose that the politician re-selects the bureaucrat if and only if $s = 1$. In this case, the bureaucrat's problem and therefore its solution are the same as in Lemma 2, except for the fact that $\mu = 1$ (as the bureaucrat knows that they are of high ability).

In the first three cases, the politician sets the tax level at the optimal formal policy level, $\tau = \Psi$ (see first claim in the proof of Lemma 3). In the fourth case, the politician sets the tax in the same way as in Propositions 1 and 2, depending on the value of ν (where both the optimal tax and the thresholds $\bar{\nu}$ and $\underline{\nu}$ incorporate the fact that $\mu = 1$). While the politician weakly prefers the fourth equilibrium (strictly whenever an informal policy is optimal), all four cases can be sustained as equilibria since the politician cannot commit to a re-selection rule. □

Next, we show that the equilibrium that corresponds to the fourth case above breaks down when we introduce small perturbation to the politician's preferences. In particular, suppose that the politician's utility in the second period is now:

$$V_2(r) = \begin{cases} \lambda F(\tau) - \tau + \varepsilon & \text{if } r = 1 \\ \lambda F(\tau) - \tau & \text{if } r = 0 \end{cases}$$

when re-selecting the incumbent, where $\varepsilon \neq 0$, but can be either positive or negative and very close to zero.[35]

Proposition 7. *For any $\varepsilon \neq 0$, the bureaucrat chooses $e_\theta(\tau) = 0$ for any $\theta \in \{D, H\}$ in any equilibria.*

Proof of Proposition 7. If $\varepsilon > 0$, then the politician's expected utility from re-selecting the bureaucrat, $\lambda F(\tau) - \tau + \varepsilon$, is strictly greater than her utility from replacing him, $\lambda F(\tau) - \tau$. As a result, there cannot be any equilibria in which the bureaucrat is not re-selected following some realizations of the signal $s \in \{0, 1\}$. In particular, the strategy of re-selecting the

[35]A positive ε captures a small preference for re-selecting the first-period bureaucrat (e.g., if there is a cost of firing them). A negative ε captures a small preference for replacing the first-period bureaucrat (e.g., if the politician has a personal connection with a bureaucrat from the replacement pool).

bureaucrat if and only if $s = 1$ (case 3 in the proof of Proposition 6) is not sequentially rational for the politician. Since this was the only possible equilibrium in which the bureaucrat could possibly choose $e^*_\theta(\tau) > 0$ in equilibrium, then we can conclude that there are no equilibria in which $e^*_\theta(\tau) > 0$. Similarly, if $\varepsilon < 0$, the politician's utility from re-selecting the bureaucrat is strictly lower than her utility from not re-selecting him for any $s \in \{0, 1\}$. As a result, there cannot be any equilibria in which the bureaucrat is re-selected following some realization of the signal s, which rules out the strategy of re-selecting the bureaucrat if and only if $s = 1$, the only possible equilibrium in which the bureaucrat could choose $e^*_\theta(\tau) > 0$. □

Finally, we show that, even if the politician has a strict preference for selecting honest bureaucrats, $e^*_\theta(\tau) = 0$ for any $\tau \in [0, +\infty)$ in any equilibria. Suppose that bribe-taking is now possible in the second period. It is straightforward to show that, in the second period, an honest bureaucrat would choose $b_H(\tau) = 0$, a dishonest bureaucrat would choose $b_D(\tau) = c^{-1}(1, D)$, and both types would choose $e^*_\theta(\tau) = 0$. Given this behavior, the politician's expected utility in the second period is: $\tilde{V}(r) = \lambda F(\tau) - \tau - \eta \mathbb{E}_\theta [b_\theta(\tau) \mid r]$.

Proposition 8. *Suppose that bribe-taking is possible in the second period, then any equilibria features no funding, $e^*_\theta(\tau) = 0$, in either period.*

Proof of Proposition 8. Recall that both funding and bribe-taking are unobservable to the politician. The politician can therefore only condition her re-selection decision on the signal s she observes and her conjecture about the level of funding of different types of bureaucrats.

Claim 1: In the second period, the politician sets $\tau^*_2 = \Psi$.

Proof. The benefit and cost of increasing tax in the second period is independent of the availability of bribes in the second period. Given assumption 2, $\lambda f(\tau) > 1$ for any $\tau \in [0, \Psi]$. In addition, the politician's expected utility is $\lambda - \tau$ for any $\tau \in [\Psi, +\infty]$. The optimal level of tax is therefore $\tau^*_2 = \Psi$. □

Claim 2: Given the politician's expected utility in period 2, she strictly prefers to re-select the politician if and only if she believes the bureaucrat is strictly less likely to be a dishonest bureaucrat than a randomly-selected bureaucrat: $r = 1$ if and only if $\mathbb{P}(\theta = D \mid s) < \nu$.

Proof. The politician's expected utility from re-selecting the bureaucrat is: $\tilde{V}(r = 1) = \lambda F(\Psi) - \tau - \eta\left[\mathbb{P}(\theta = H \mid s) \times 0 + \mathbb{P}(\theta = D \mid s) b_H(\tau)\right]$. Her expected utility from replacing the bureaucrat is: $\tilde{V}(r = 0) = \lambda F(\Psi) - \tau - \eta\left[(1 - \nu) \times 0 + \nu b_H(\tau)\right]$. So she strictly prefers re-selecting if and only if: $\tilde{V}(r = 1) = \lambda - \tau - \eta \mathbb{P}(\theta = D \mid s) b_H(\tau) > \tilde{V}(r = 0) = \lambda - \tau - \eta \nu b_H(\tau)$, or equivalently, $\mathbb{P}(\theta = D \mid s) < \nu$. □

Turning attention to the first period, there are three classes of possible equilibria (semi-separating equilibria are ruled out because we only allow pure strategies). Note that, since the politician cannot observe bribes, the only bureaucrat choice relevant for the politician's beliefs about the bureaucrat's honesty is his equilibrium choice of funding $e^*_\theta(\tau)$.

1. **Separating equilibria in which the dishonest bureaucrat funds more.** Consider equilibria in which $e^*_D(\tau) > e^*_H(\tau)$ for any $\tau \in [0, +\infty)$. In this case, $\mathbb{P}(s = 1 \mid \theta = D) = F(\tau + e^*_D(\tau)) > \mathbb{P}(s = 1 \mid \theta = H) = F(\tau + e^*_H(\tau))$. As a result, the politician's posterior belief about the bureaucrat's honesty is:

$$\mathbb{P}(\theta_D \mid s = 1) = \frac{\mathbb{P}(s = 1 \mid \theta = D)\nu}{\mathbb{P}(s = 1 \mid \theta = D)\nu + \mathbb{P}(s = 1 \mid \theta = H)(1 - \nu)} > \nu$$

   The politician therefore re-selects the bureaucrat if and only if $s = 0$. Given this re-selection rule, the bureaucrat's objective function given some first-period funding e and bribe b and second period bribe $b_{2,\theta}$ is $u(e, b \mid \theta) = w_1 + (w_2 + b_{2,\theta}(\tau_2))(1 - \phi F(\tau + e)) + b - e - C(b, \theta)$. Since this function is everywhere decreasing in e, the solution to this problem is $e_\theta(\tau) = 0$, $b_H(\tau) = 0$ and $b_D(\tau) = c^{-1}(1, \theta)$ for any $\tau \in [0, 1 + \infty)$.

   As a result, $e^*_D(\tau) = e^*_H(\tau) = 0$, which contradicts the premise that $e^*_D(\tau) > e^*_H(\tau)$. This class of equilibria therefore cannot exist.

2. **Separating equilibria in which the honest bureaucrat funds more.** Consider equilibria in which $e^*_H(\tau) > e^*_D(\tau)$ for any $\tau \in [0, +\infty)$. In this case, $\mathbb{P}(s = 1 \mid \theta = D) = F(\tau + e^*_D(\tau)) < \mathbb{P}(s = 1 \mid \theta = H) = F(\tau + e^*_H(\tau))$. As a result, the politician's posterior belief about the bureaucrat's honesty is:

$$\mathbb{P}(\theta_D \mid s = 1) = \frac{\mathbb{P}(s = 1 \mid \theta = D)\nu}{\mathbb{P}(s = 1 \mid \theta = D)\nu + \mathbb{P}(s = 1 \mid \theta = H)(1 - \nu)} < \nu$$

   The politician's re-selection rule is therefore to re-select the bureaucrat if and only if $s = 1$. Given this re-selection rule, the bureaucrat's objective function is $u(e, b \mid \theta) = w_1 + (w_2 + b_{2,\theta}(\tau_2))\phi F(\tau + e) + b - e - C(b, \theta)$. We can solve this problem by following the steps in Lemma 2. The only difference with Lemma 2 is that, in addition to having a lower marginal cost of taking bribes, the dishonest bureaucrat also has a higher marginal benefit of being re-selected than the honest bureaucrat as $w_2 + b_{2,\theta}(\tau_2) = w_2 + c^{-1}(1, D) > w_2$. We can therefore show that, in this case too, the dishonest bureaucrat's funding is weakly higher than the honest bureaucrat's funding for any $\tau \in [0, +\infty)$. As a result, $e^*_D(\tau) \geq e^*_H(\tau)$, which contradicts the premise that $e^*_H(\tau) > e^*_D(\tau)$. This class of equilibria therefore cannot exist.

3. **Pooling equilibria.** The final possible class of equilibria are ones in which both types of bureaucrats fund the same amount of public services: $e^*_D(\tau) = e^*_H(\tau)$, for any $\tau \in [0, +\infty)$. In this case, $\mathbb{P}(s = 1 \mid \theta = D) = F(\tau + e^*_D(\tau)) = \mathbb{P}(s = 1 \mid \theta = H) = F(\tau + e^*_H(\tau))$. As a result, the politician's posterior belief about the bureaucrat's honesty is: $\mathbb{P}(\theta_D \mid s = 1) = \frac{\mathbb{P}(s=1|\theta=D)\nu}{\mathbb{P}(s=1|\theta=D)\nu+\mathbb{P}(s=1|\theta=H)(1-\nu)} = \nu$. The politician is therefore indifferent between re-selecting and replacing the bureaucrat.

   Following the logic of the proof of Proposition 7, the equilibrium funding is $e^*_\theta = 0$ for any $\theta \in \{H, D\}$ when (1) the politician re-selects the bureaucrat for any $s \in \{0, 1\}$, (2) the politician replaces the bureaucrat for any $s \in \{0, 1\}$, or (3) the politician re-selects the bureaucrat if and only if $s = 0$. When the politician re-selects the bureaucrat if and only if $s = 1$, the bureaucrat's objective function is the same as in case 2 above. As a result, the dishonest bureaucrat would provide a strictly larger amount of funding than the honest bureaucrat, thus contradicting the premise that $e^*_H(\tau) = e^*_D(\tau)$ for any $\tau \in [0, +\infty)$.

As a result, the only possible equilibria are pooling equilibria in which (1) $e^*_D(\tau) = e^*_H(\tau) = 0$, for any $\tau \in [0, +\infty)$, (2) $b^*_H(\tau) = 0$, $b^*_D(\tau) = c^{-1}(1, D)$, for any $\tau \in [0, +\infty)$, and (3) the politician either never re-selects the bureaucrat, always re-selects the bureaucrat, or re-selects the bureaucrat if and only if $s = 0$. □

A.5 Collusion between the bureaucrat and the politician

To analyze the role of collusion between the bureaucrat and the politician, we consider the following model. Suppose that the bureaucrat is required by the politician to transfer a proportion $\pi \in (0, 1)$ of every rupee of bribe he takes back to the politician. However, the bureaucrat remains punishable for the entire size of the bribe he takes. This modification changes the baseline model in two ways.

First, the bureaucrat's problem becomes the following:

$$\max_{b,e} \quad w_1 + (1-\pi)b - e + \mu\phi w_2 F(\tau + e) - C(b,\theta) \quad \text{s.t.} \quad 0 \le e \le w_1 + (1-\pi)b,\ 0 \le b$$

Second, the politician's per-period payoff function now includes an extra term which captures the portion of bribes obtained from the bureaucrat:

$$v_t(y_t, \tau_t, b_t) = \begin{cases} \lambda - \tau_t - \eta b_t + \pi b_t & \text{if } y_t \ge \bar{y} \\ -\tau_t - \eta b_t + \pi b_t & \text{if } y_t < \bar{y} \end{cases}$$

To reduce the number of cases, we assume that $\frac{1}{\mu w_2 f(w_1)} < \phi < \frac{1}{\mu w_2 f(\Psi)}$ and $\mu\phi w_2 f(\Psi) \ge c(\Psi - w_1, D)$, which implies that the bureaucrat never chooses a level of funding sufficient to guarantee that the citizens' needs will be met but that there are some values of τ for which the honest bureaucrat would want to take additional bribes (i.e., Case 2 of the proof of Lemma 3). In addition, we assume that the marginal cost of taking bribes for the dishonest type at $b = 0$ is low enough that the dishonest bureaucrat always wants to take some bribes no matter the share it has to redistribute to the politician: $c(0, D) < 1 - \pi$. We also change the normalization of the honest bureaucrat's marginal cost at $b = 0$ to $c(0, H) = 1 - \pi$ to make it consistent with our baseline case. Finally, we focus on the case where the share of dishonest bureaucrats is high enough that the politician prefers an informal policy with high corruption to one with low corruption (the relevant threshold on that share is defined formally below).

In this alternative model and under these assumptions, we show the following result:

Proposition 9. *A higher portion of bribes shared with the politician, π, weakly decreases funding by the bureaucrats for any given level of tax and can either make informal fiscal systems more or less desirable.*

Proof of Proposition 9. We first note that Lemma 1 remains unchanged in this new model since there are no opportunities for bribes in the second period.

Next, we show that Lemma 4 becomes:

Lemma 11.

- *If $\tau \le \tau_1$, $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e + \tau) = \frac{c\left(\frac{e - w_1}{1-\pi}, \theta\right)}{1-\pi}$ and $b^*_\theta(\tau) = \frac{e^*_\theta(\tau) - w_1}{1-\pi}$, $\forall \theta \in \{H, D\}$.*

- *If* $\tau \in (\tau_1, \tau_2]$, $e^*_H(\tau)$ *solves* $\mu\phi w_2 f(e^*_H(\tau)+\tau) = \frac{c\left(\frac{e^*_H(\tau)-w_1}{1-\pi}, H\right)}{1-\pi}$ *and* $b^*_H(\tau) = \frac{e^*_H(\tau)-w_1}{1-\pi}$ *while* $e^*_D(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ *and* $b^*_D(\tau) = c^{-1}(1-\pi, D)$.

- *If* $\tau \in (\tau_2, \tau_3]$, $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$, $\forall\theta \in \{H, D\}$, $b^*_D(\tau) = c^{-1}(1-\pi, D)$, *and* $b^*_H(\tau) = 0$.

- *If* $\tau \geq \tau_3$, $e^*_\theta(\tau) = 0$, $\forall\theta \in \{H, D\}$, $b^*_D(\tau) = c^{-1}(1-\pi, D)$, $b^*_H(\tau) = 0$.

Where $\tau_1 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D)$, $\tau_2 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1$, and $\tau_3 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$.

Proof of Lemma 11. We build on the proof of Lemma 4 but incorporate the additional $(1-\pi)$ term in the budget constraint and the objective function. Given a tax rate τ and the politician's retention rule from Lemma 1, the bureaucrat's best response solves:

$$\max_{b,e} \quad w_1 + (1-\pi)b - e + \mu\phi w_2 F(\tau+e) - C(b,\theta) \quad \text{s.t.} \quad 0 \leq e \leq w_1 + (1-\pi)b,\ 0 \leq b$$

The Lagrangian is:

$$\mathcal{L}(e, b; \gamma) = w_1 + (1-\pi)b - e + \mu\phi w_2 F(\tau+e) - C(b,\theta) + \gamma(w_1 + (1-\pi)b - e)$$

Where γ is the Lagrange multiplier. The first-order conditions are:

$$\frac{\partial\mathcal{L}(e,b)}{\partial e} = -1 + \mu\phi w_2 f(\tau+e) - \gamma = 0$$
$$\frac{\partial\mathcal{L}(e,b)}{\partial b} = 1 - \pi - c(b,\theta) + \gamma(1-\pi) = 0$$

The second-order condition is satisfied since F is concave and C is convex (so $-C(b,\theta)$ is concave). There are two cases:

1. **Case 1:** If the constraint does not bind, then by complementary slackness $\gamma = 0$ and the first-order condition with respect to e gives $\mu\phi w_2 f(\tau + e^*_\theta) - 1 = 0$.

   (a) If $\mu\phi w_2 f(\tau) - 1 < 0$, then $\mu\phi w_2 f(\tau+e) - 1 < 0$ for any $e \in [0, \Psi - \tau]$. Since $f(\tau + e) = 0$ for $e > \Psi - \tau$, then $\mu\phi w_2 f(\tau+e) - 1 = -1 < 0$ for $e > \Psi - \tau$. The objective function is therefore everywhere decreasing in e and the unconstrained optimal is $e^* = 0$. Since the constraint does not bind at $e^* = 0$, this is also the constrained optimal.

(b) If $\mu\phi w_2 f(\tau) - 1 \geq 0$, then the first-order condition is satisfied for some $e^* \in [0, \Psi - \tau]$ such that $\mu\phi w_2 f(\tau + e^*) = 1$. The unconstrained optimal is therefore $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. Recall that we assume that $\mu\phi w_2 f(\Psi) - 1 < 0$ throughout, so these are the only two possible cases.

We now turn to the optimal unconstrained bribe. If the budget constraint is not binding ($\gamma = 0$), the first-order condition with respect to b gives $c(b^*_D, D) = 1 - \pi$ for type D but is never satisfied for type H since $c(b, H) > c(0, H) = 1 - \pi$ for any $b > 0$ (by convexity of C). The budget constraint is therefore binding if $e^* \geq w_1 + (1-\pi)c^{-1}(1-\pi, D)$ for $\theta = D$ and if $e^* \geq w_1$ for $\theta = H$. Solving for the constrained optima:

(a) If $\mu\phi w_2 f(\tau) - 1 < 0$, the constraint never binds so the constrained optimal personal funding is $e^*_\theta(\tau) = 0$ as described above.

(b) If $\mu\phi w_2 f(\tau) - 1 \geq 0$, then the unconstrained optimal private funding is $e^* = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$, so the budget constraint is satisfied if $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau < w_1$ for $\theta = H$, and $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau < w_1 + (1-\pi)c^{-1}(1-\pi, D)$ for $\theta = D$. When these constraints are satisfied, the constrained optimal personal funding is therefore $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$.

2. **Case 2:** If the solution above violates the budget constraint, then the budget constraint must bind at the optimal level of funding and bribe, so $\gamma > 0$. We can substitute the bribe into the bureaucrat's problem by using the binding constraint: $e = w_1 + (1-\pi)b$ or, equivalently, $b = \frac{e - w_1}{1-\pi}$. Substituting in the first-order conditions and solving them simultaneously gives

$$\mu\phi w_2 f(\tau + e) = 1 + \gamma = \frac{c\left(\frac{e-w_1}{1-\pi}, \theta\right)}{1-\pi}$$

We can use the intermediate value theorem to show that there exists a value of e that solves $\mu\phi w_2 f(\tau + e) = \frac{c\left(\frac{e-w_1}{1-\pi}, \theta\right)}{1-\pi}$. Let $LHS(e) = \mu\phi w_2 f(e + \tau)$ and $RHS(e) = \frac{c\left(\frac{e-w_1}{1-\pi}, \theta\right)}{1-\pi}$. First note that, following the proof of Lemma 4, $LHS(e)$ is decreasing in e and $RHS(e)$ is increasing in e. We therefore need to show that $LHS(e) > RHS(e)$ at the smallest value of e and $LHS(e) < RHS(e)$ at the largest value of e.

- **At the smallest value of e such that the constraint binds**:

  For $\theta = H$, the lowest value of e such that the constraint binds is $e = w_1$. At

$e = w_1$, we have that, $\forall \tau \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1\right]$, $LHS(e) = \mu\phi w_2 f(w_1 + \tau) > \mu\phi w_2 f\left(w_1 + f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1\right) = \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) = 1 > 1 - \pi = c(0, H) = RHS(e)$.

For $\theta = D$, the lowest value of e such that the constraint binds is $e = w_1 + (1-\pi)c^{-1}(1-\pi, D)$. At $e = w_1 + (1-\pi)c^{-1}(1-\pi, D)$, we have that, $\forall \tau \in \left[0, f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D)\right]$,

$$
\begin{aligned}
LHS(e) &= \mu\phi w_2 f\left(w_1 + (1-\pi)c^{-1}(1-\pi, D) + \tau\right) \\
&> \mu\phi w_2 f\left(w_1 + (1-\pi)c^{-1}(1-\pi, D) + f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D)\right) \\
&= \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right)\right) \\
&= 1 \\
&= \frac{1}{1-\pi} c(c^{-1}(1-\pi, D), D) \\
&= \frac{1}{1-\pi} c\left(\frac{w_1 + (1-\pi)c^{-1}(1-\pi, D) - w_1}{1-\pi}, D\right) = RHS(e)
\end{aligned}
$$

Therefore, for both $\theta \in \{H, D\}$, $LHS(e) > RHS(e)$ at the smallest value of e.

- **At the largest possible value of** e: Since $\mu\phi w_2 f(\Psi) - 1 \leq 0$, the largest possible value of e is the unconstrained optimal $e = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$. At this value, $LHS(e) = \mu\phi w_2 f\left(f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau + \tau\right) = 1$ and $RHS(e) = \frac{1}{1-\pi} c\left(\frac{f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1}{1-\pi}, \theta\right)$.

  For type $\theta = H$, we have that $RHS(e) = \frac{1}{1-\pi} c\left(\frac{f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1}{1-\pi}, H\right) > \frac{c(0,H)}{1-\pi} = \frac{1-\pi}{1-\pi} = 1$ since c is increasing and since $c(0, H) = 1 - \pi$, so $RHS(e) > 1 = LHS(e)$. Similarly, for type $\theta = D$, we have $\frac{1}{1-\pi} c\left(\frac{f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1}{1-\pi}, D\right) > 1$. This follows from the fact that the constraint is binding at $e = f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau$, so that $f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau > w_1 + (1-\pi)c^{-1}(1-\pi, D)$, which is equivalent to

$$
\frac{1}{1-\pi} c\left(\frac{f^{-1}\left(\frac{1}{\mu\phi w_2}\right) - \tau - w_1}{1-\pi}, D\right) > 1
$$

  Therefore, for both $\theta \in \{H, D\}$, $RHS(e) > LHS(e)$ at the smallest value of e.

s Since $LHS(e)$ is decreasing in e and $RHS(e)$ is increasing in e, $LHS(e) > RHS(e)$ at the smallest value of e and $LHS(e) < RHS(e)$ at the largest value of e, then by the intermediate value theorem, there exists $e^*_\theta(\tau) \in [w_1 + \mathbb{1}\{\theta = D\}(1-\pi)c^{-1}(1-\pi, D), \Psi - \tau]$ such that $LHS(e^*_\theta(\tau)) = RHS(e^*_\theta(\tau))$.

We can therefore conclude that,

1. If $\tau \leq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D) = \tau_1$, the budget constraint of both types binds. For both types θ, $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e^*_\theta + \tau) = \frac{c\left(\frac{e^*_\theta - w_1}{1-\pi}, \theta\right)}{1-\pi}$ and $b^*_\theta(\tau) = \frac{e^*_\theta(\tau) - w_1}{1-\pi}$.
2. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D), f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right]$, i.e., $\tau \in (\tau_1, \tau_2]$, the budget constraint of the honest type binds but not that of the dishonest type. The honest type's private funding and bribe solve $\mu\phi w_2 f(e^*_H + \tau) = \frac{c\left(\frac{e^*_H - w_1}{1-\pi}, H\right)}{1-\pi}$ and $b^*_H(\tau) = \frac{e^*_H(\tau) - w_1}{1-\pi}$. The dishonest type's funding and bribe are: $e^*_D(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ and $b^*_D(\tau) = c^{-1}(1-\pi, D)$.
3. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1, f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right]$, i.e., $\tau \in (\tau_2, \tau_3]$, neither types' budget constraint binds so $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$, $b^*_D(\tau) = c^{-1}(1-\pi, D)$ and $b^*_H(\tau) = 0$.
4. If $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) = \tau_3$, then $e^*_\theta(\tau) = 0$, $b^*_\theta(D) = c^{-1}(1-\pi, D)$, $b^*_\theta(H) = 0$.

□

The first statement in Proposition 9 follows directly from Lemma 11.

1. If $\tau \leq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - c^{-1}(1, D)$, then $\tau \leq f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D)$ so $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e^*_\theta + \tau) = \frac{c\left(\frac{e^*_\theta - w_1}{1-\pi}, \theta\right)}{1-\pi}$ when $\pi > 0$ and $e^*_\theta(\tau)$ solves $\mu\phi w_2 f(e^*_\theta + \tau) = c\left(e^*_\theta - w_1, \theta\right)$ when $\pi = 0$. Since $\frac{1}{1-\pi} > 1$, the marginal cost of funding (the right-hand side of the equation) is higher when $\pi > 0$, so the value of e^* that solves the equation must be lower.
2. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - c^{-1}(1, D), f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D)\right]$, then the honest bureaucrat's funding is the same in both scenarios as in the first case above so the same logic applies. However, the dishonest bureaucrat's funding is now $e^*_D(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ when $\pi = 0$ but solves $\mu\phi w_2 f(e^*_D + \tau) = \frac{c\left(\frac{e^*_D - w_1}{1-\pi}, D\right)}{1-\pi}$ when

$\pi > 0$. Since $f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ is also the unconstrained optimal when $\pi > 0$ but cannot be attained when $\pi > 0$ since the dishonest bureaucrat's budget constrain is binding, then we must have that $e^*_D(\tau)$ when $\pi > 0$ is lower than $f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ which is the funding when $\pi = 0$.

3. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D), f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1\right]$, then the dishonest type's funding is $e^*_D(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ both when $\pi > 0$ and when $\pi = 0$. The honest type's funding in each scenario is the same as in the two previous cases, so the funding is strictly lower when $\pi > 0$ for the honest type and the same for any π for the dishonest type.

4. If $\tau \in \left(f^{-1}\left(\frac{1}{\phi w_2}\right) - w_1, f^{-1}\left(\frac{1}{\phi w_2}\right)\right]$, then $e^*_\theta(\tau) = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - \tau$ for both types in both scenarios so the funding is independent of π.

5. If $\tau \geq f^{-1}\left(\frac{1}{\phi\mu w_2}\right)$, then $e^*_\theta(\tau) = 0$ for both types in both scenarios so the funding is independent of π.

The next step is to show that we can obtain a modified version of Lemma 3 to define a threshold on the share of dishonest bureaucrat above which the politician prefers an informal policy with high corruption.[36]

Lemma 12. *The politician prefers an informal policy with high corruption to one with low corruption if $\nu \geq \bar{\nu}$ and vice versa if $\nu \leq \underline{\nu}$, where the thresholds are given by:*

$$\bar{\nu} = \max_{\tau\in[0,\tau_2]} \left\{ \frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \left(\frac{\eta-\pi}{1-\pi}\right)\frac{\partial e^*_H(\tau)}{\partial\tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \left(\frac{\eta-\pi}{1-\pi}\right)\frac{\partial e^*_H(\tau)}{\partial\tau}} \right\}$$

$$\underline{\nu} = \min_{\tau\in[0,\tau_2]} \left\{ \frac{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \left(\frac{\eta-\pi}{1-\pi}\right)\frac{\partial e^*_H(\tau)}{\partial\tau} - 1}{\mu U_2 f(\tau + e^*_H(\tau))\left(1 + \frac{\partial e^*_H(\tau)}{\partial\tau}\right) - \left(\frac{\eta-\pi}{1-\pi}\right)\frac{\partial e^*_H(\tau)}{\partial\tau}} \right\}$$

Proof of Lemma 12. First note that the first claim in the proof of Lemma 3 continues to hold as it is independent of π. In the first step of the proof, we can focus on Case 2 as we have

[36]We restrict attention to the case where $\tau_1 = f^{-1}\left(\frac{1}{\phi\mu w_2}\right) - w_1 - (1-\pi)c^{-1}(1-\pi, D) < 0$ to be consistent with Lemma 3.

excluded the other cases. The politician's expected intertemporal payoff becomes:

$$V(\tau) = \nu \left[\mu F \left(f^{-1} \left(\frac{1}{\phi \mu w_2} \right) \right) U_2 - (\eta - \pi) c^{-1}(1-\pi, D) - \tau \right] + (1-\nu) \left[\mu F(\tau + e_H^*(\tau)) U_2 - (\eta - \pi) \frac{e_H^*(\tau) - w_1}{1-\pi} - \tau \right] + \mu\lambda - \Psi$$

The derivative of $V(\tau)$ with respect to τ for $\tau \in [0, \tau_2]$ is:

$$\frac{\partial V(\tau)}{\partial \tau} = \nu(-1) + (1-\nu) \left[\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau} \right) - (\eta - \pi) \frac{1}{1-\pi} \frac{\partial e_H^*(\tau)}{\partial \tau} - 1 \right]$$

This derivative is positive if and only if:

$$\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau} \right) - \left(\frac{\eta - \pi}{1-\pi} \right) \frac{\partial e_H^*(\tau)}{\partial \tau} - 1 \geq \nu \left(\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau} \right) - \left(\frac{\eta - \pi}{1-\pi} \right) \frac{\partial e_H^*(\tau)}{\partial \tau} \right)$$

Next, we note that Lemma 6 continues to hold as introducing π only re-scales the terms but does not change their signs. Finally, as in the proof of Lemma 3, $\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \left(\frac{\eta-\pi}{1-\pi}\right) \frac{\partial e_H^*(\tau)}{\partial \tau} - 1 > 0$ since the only difference with the proof of Lemma 14 is that η is replaced by $\frac{\eta-\pi}{1-\pi}$, and that $\frac{\eta-\pi}{1-\pi} \geq 1$ given that $\eta \geq 1 > \pi$. The thresholds $\underline{\nu}$ and $\bar{\nu}$ are therefore well-defined. The final steps of the proof also follow straightforwardly from the proof of Lemma 3. □

Given the results above, we now prove the second statement of Proposition 9. Suppose that $\nu \geq \bar{\nu}$ so the first segment of the politician's objective function, $V(\tau)$, is decreasing. Following the logic of the proof of Case 1 of Proposition 1, the politician prefers an informal fiscal system if $V(0) > V(\Psi)$. Here, this requires:

$$\nu \left[\mu U_2 F \left(f^{-1} \left(\frac{1}{\phi \mu w_2} \right) \right) - (\eta - \pi) c^{-1}(1-\pi, D) \right] + (1-\nu) \left[\mu U_2 F(e_H^*(0)) - (\eta - \pi) \frac{e_H^*(0) - w_1}{1-\pi} \right] + \mu\lambda - \Psi > \mu U_2 F(\Psi) - \Psi - \nu(\eta - \pi) c^{-1}(1-\pi, D) + \mu\lambda - \Psi$$

Or equivalently,

$$\nu\left[\mu U_2 F\left(f^{-1}\left(\frac{1}{\phi\mu w_2}\right)\right)\right]+(1-\nu)\left[\mu U_2 F(e_H^*(0))-(\eta-\pi)\frac{e_H^*(0)-w_1}{1-\pi}\right]>\mu U_2-\Psi \quad (17)$$

The right-hand side of inequality (17) is independent of π, while the left-hand side can either increase or decrease in π since

$$\frac{\partial LHS(\pi)}{\partial\pi}=(1-\nu)\left[\frac{\partial e_H^*(0)}{\partial\pi}\left(\mu U_2 f(e_H^*(0))-\left(\frac{\eta-\pi}{1-\pi}\right)\right)-\frac{\eta-1}{(1-\pi)^2}(e_H^*(0)-w_1)\right]$$

can be either positive or negative. Indeed, we know that $\frac{\partial e_H^*(0)}{\partial\pi}<0$ given the condition that implicitly defines $e_H^*(0)$, that $\mu U_2 f(e_H^*(0))>1$ given assumption 2, that $\frac{\eta-\pi}{1-\pi}>1$ and $\frac{\eta-1}{(1-\pi)^2}(e_H^*(0)-w_1)>0$ since $\eta>1$. Therefore, if $\mu U_2 f(e_H^*(0))>\frac{\eta-\pi}{1-\pi}$ (e.g., if ϕ is large enough) then $\frac{\partial e_H^*(0)}{\partial\pi}\left(\mu U_2 f(e_H^*(0))-\left(\frac{\eta-\pi}{1-\pi}\right)\right)<0$ so $\frac{\partial LHS(\pi)}{\partial\pi}<0$. Instead, if $\mu U_2 f(e_H^*(0))<\frac{\eta-\pi}{1-\pi}$ (e.g., if ϕ is small enough), then $\frac{\partial e_H^*(0)}{\partial\pi}\left(\mu U_2 f(e_H^*(0))-\left(\frac{\eta-\pi}{1-\pi}\right)\right)>0$ and it is possible for $\frac{\partial LHS(\pi)}{\partial\pi}>0$ (e.g., if $\mu U_2 f(e_H^*(0))$ is close to 1 and η is large). □

A.6 Patronage

To analyze the role of patronage, we consider the following model. Suppose that the politician can be personally connected to some bureaucrats. When a politician has a personal connection with the bureaucrat, she receives a benefit, E, (e.g. electoral support) in exchange for retaining the bureaucrat. This implies that the politician might retain the bureaucrat for the second period independently of the bureaucrat's performance.

A.6.1 Connection with incumbent bureaucrat

We begin by showing that, when the politician is connected to a bureaucrat in office, the bureaucrat's incentives to provide funding decrease and informal fiscal systems becomes less desirable.

Selection rule. We first show how the selection rule changes when the politician is connected to the bureaucrat in office. Let

$$\bar{E}=\left(\mu\lambda F(\tau_2^*(r=0))-\tau_2^*(r=0)\right)-\left(\frac{(1-\phi F(\Psi)\mu}{(1-\phi F(\Psi)\mu+(1-\phi F(0))(1-\mu)}\lambda F(\tau_2^*(r=1))-\tau_2^*(r=1)\right)$$

Lemma 13. *When the bureaucrat is not connected to the politician, or is connected to the politician but $E < \bar{E}$, the bureaucrat is re-selected if and only $s = 1$. If $E \geq \bar{E}$, a connected bureaucrat is always re-selected for any $s \in \{0, 1\}$.*

Proof. An unconnected bureaucrat faces the same selection rule as in Lemma 1 since there is no additional benefit of retaining them. When facing a connected bureaucrat, the politician's utility from re-selecting him, following some signal $s \in \{0, 1\}$ is:

$$E + \mathbb{P}(\omega = 1 \mid s)(\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)) + \mathbb{P}(\omega = 0 \mid s)(-\tau_2^*(r = 1))$$

If $s = 1$, then $\mathbb{P}(\omega = 1 \mid s = 1) = 1$ as shown in the proof of Lemma 1, so the expected payoff from re-selecting is: $E + \lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)$. Instead, replacing the bureaucrat gives a payoff of $\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$. The politician therefore retains the bureaucrat since: $E + \lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) > \mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$.

If $s = 0$, then as shown in the proof of Lemma 1,

$$\begin{aligned}\mathbb{P}(\omega = 1 \mid s = 0) = \mathbb{P}(\theta = H \mid s = 0) &\frac{(1 - \phi F(\tau_1 + e_1^*(H)))\mu}{(1 - \phi F(\tau_1 + e_1^*(H)))\mu + (1 - \phi F(0))(1 - \mu)} \\ + \mathbb{P}(\theta = D \mid s = 0) &\frac{(1 - \phi F(\tau_1 + e_1^*(D)))\mu}{(1 - \phi F(\tau_1 + e_1^*(D)))\mu + (1 - \phi F(0))(1 - \mu)} \quad < \mu\end{aligned}$$

So the expected payoff from re-selecting is: $E + \mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)$. Instead, replacing the bureaucrat gives a payoff of $\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)$. The politician therefore retains the bureaucrat if and only if:

$$\begin{aligned} & E + \mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1) > \mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0) \\ \Leftrightarrow \quad & E > \big(\mu\lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)\big) - \big(\mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1)) - \tau_2^*(r = 1)\big)\end{aligned}$$

Note that $\mathbb{P}(\omega = 1 \mid s = 0)$ is a function of the bureaucrat's funding so is determined in equilibrium. However, we can obtain a sufficient condition on the primitives for the condition to hold given some equilibrium conjecture on the bureaucrat's funding. We show in the next part of the proof that, if the bureaucrat is always re-selected, then the optimal funding is $e_\theta^* = 0$ as deviating would decrease the bureaucrat's perceived ability and come at a cost to the bureaucrat. We can therefore evaluate the condition above at $e_\theta^* = 0$. In addition, the probability $\mathbb{P}(\omega = 1 \mid s = 0)$ is decreasing in τ. As a result, the politician might choose not to re-select after observing $s = 0$ when τ is high. A sufficient condition for the politician to re-select the connected bureaucrat after $s = 0$, given that $e_\theta^* = 0$, is that even at the highest possible level of tax, $\tau = \Psi$, the politician still prefers re-selecting a connected politician following $s = 0$:

$$
\begin{aligned}
&E > \big(\mu\lambda F(\tau_2^*(r=0)) - \tau_2^*(r=0)\big) - \big(\mathbb{P}(\omega = 1 \mid s = 0, e_H = 0, e_D = 0, \tau = \Psi)\lambda F(\tau_2^*(r=1)) - \tau_2^*(r=1)\big) \\
\Leftrightarrow \quad &E > \big(\mu\lambda F(\tau_2^*(r=0)) - \tau_2^*(r=0)\big) - \left(\frac{(1-\phi F(\Psi)\mu}{(1-\phi F(\Psi)\mu + (1-\phi F(0))(1-\mu)}\lambda F(\tau_2^*(r=1)) - \tau_2^*(r=1)\right)
\end{aligned}
$$

□

Bureaucrat's funding. A bureaucrat who is not connected faces the same re-selection rule as in the baseline model. As a result, his funding and bribe choice is the same as in Lemma 2. For a connected bureaucrat, there are two cases to consider.

- If $E < \bar{E}$, a connected bureaucrat also faces the same retention rule as in the baseline model and therefore chooses funding and bribes as in Lemma 2.
- If $E \geq \bar{E}$, the bureaucrat is re-selected for any $s \in \{0, 1\}$. Following the logic of case 1 in the proof of Proposition 7, a connected bureaucrat would therefore never fund public services: $e_\theta^* = 0$, $\forall \theta \in \{D, H\}$.

Politician's choice of tax. If the politician faces an unconnected bureaucrat or if she faces a connected bureaucrat but $E < \bar{E}$, her choice of τ is the same as in Proposition 1 or 2. Instead, if she faces a connected bureaucrat and $E \geq \bar{E}$, then her problem becomes:

$$
\begin{aligned}
\max_{\tau} V(\tau) =& \mu\left(F(\tau)\lambda - \tau - \eta\nu c^{-1}(1, D) + \lambda - \Psi\right) + (1-\mu)\left(0 - \tau - \eta\nu c^{-1}(1, D)\right) \\
=& \mu F(\tau)[\lambda + \lambda - \Psi] - \tau - \eta\nu c^{-1}(1, D)
\end{aligned}
$$

The first order condition is $\mu(2\lambda - \Psi)f(\tau) - 1$. Given assumption 2, $\mu\lambda f(\Psi) - 1 > 0$, and since $\lambda > \Psi$, $\mu(2\lambda - \Psi)f(\Psi) - 1 = \mu\lambda f(\Psi) + \mu(\lambda - \Psi)f(\Psi) - 1 > \mu\lambda f(\Psi) - 1 > 0$. The objective function is therefore increasing everywhere on $\tau \in [0, \Psi]$ so $\tau^* = \Psi$.

Comparing the optimal fiscal system with and without patronage. If the politician faces a unconnected bureaucrat or if the politician faces a connected bureaucrat but $E < \bar{E}$, the choice of fiscal system is the same with or without patronage. If she faces a connected bureaucrat and $E \geq \bar{E}$, then the optimal tax rate is $\tau^* = \Psi$ and $e^* = 0$. In other words, a formal fiscal system is always preferred when the bureaucrat is connected and $E \geq \bar{E}$.

Suppose that the politician is connected to the bureaucrat with some probability $\sigma \in (0, 1)$, we can then draw the following conclusion:

Result: a formal fiscal system is weakly more likely under patronage.

Proof. Suppose that the parameters are such that, without patronage, an informal fiscal system is optimal (following Propositions 1 or 2). Then when there is no patronage, the probability of a formal fiscal system is 0, while when there is patronage, the probability of a formal fiscal system is $\sigma > 0$. If instead, the parameters are such that, without patronage a formal fiscal system is optimal, then the probability of a formal system is 1 with or without patronage. □

A.6.2 Connection with replacement bureaucrat

We now present an example in which patronage can increase the bureaucrat's funding, when the politician is connected to a bureaucrat from the replacement pool.

We modify the model and assume that bureaucrats from the replacement pool have a lower expected ability. Let μ_C the probability that a bureaucrat is of ability $\omega = 1$ in the replacement pool, and assume that $\mu_C < \frac{(1-\phi F(\Psi))\mu}{(1-\phi F(\Psi))\mu+(1-\phi F(0))(1-\mu)}$. If the bureaucrat from the replacement pool is not connected, this assumption implies that the incumbent bureaucrat is guaranteed to be retained even if the bureaucrat provides no funding in the first period: $r = 1,\ \forall s \in \{0, 1\}$. As a result, following the logic of case 1 in the proof of Proposition 7, the incumbent bureaucrat would never fund public services: $e^*_\theta = 0,\ \forall\theta \in \{D, H\}$. Without any bureaucrat funding, the politician would therefore always strictly prefer a formal fiscal system.

Suppose instead that the bureaucrat in the replacement pool is connected. If

$$\begin{aligned}\lambda F(\tau_2^*(r = 1, s = 1)) - \tau_2^*(r = 1, s = 1) &> E + \mu_C \lambda F(\tau_2^*(r = 0)) - \tau_2^*(r = 0)\\ &> \mathbb{P}(\omega = 1 \mid s = 0)\lambda F(\tau_2^*(r = 1, s = 0)) - \tau_2^*(r = 1, s = 0),\end{aligned}$$

then the politician prefers to replace the incumbent bureaucrat following the signal $s = 0$ but not following $s = 1$. The bureaucrat's problem therefore becomes the same as in Lemma 2 and the bureaucrat provides some funding in equilibrium (as long as the conditions for the politician to choose an informal fiscal system are satisfied). As a result, patronage makes the politician more likely to choose an informal fiscal system.

A.7 Low cost of corruption, $\eta < 1$

We now analyze how our results are affected when we relax the assumption that $\eta > 1$. First note that Lemmas 1 and 2 are unaffected by this assumption. Indeed, the politician's selection decision only depends on her belief about the bureaucrat's ability given the signal she receives and does not depend on whether $\eta > 1$ or not. The bureaucrat's behavior is independent of η and therefore also not affected by this assumption. We therefore show how our main results (Lemma 3 and Propositions 1 and 2) change when $\eta < 1$.

A.7.1 Informal policies with high vs. low corruption

We first review the effect of assuming that $\eta < 1$ on Lemma 3.

Lemma 14. *If $\eta < 1$ and $\phi > \frac{1}{\mu w_2 f(w_1)}$, there exists a threshold $\bar{\eta}$ such that:*

- *If $\eta < \bar{\eta}$ the politician always prefers an informal policy with high corruption to one with low corruption.*
- *If $\eta \geq \bar{\eta}$, there exist thresholds $\bar{\nu} \in (0,1)$ and $\underline{\nu} \in (0,1]$ on the probability that a bureaucrat is dishonest such that the politician prefers an informal policy with high corruption to one with low corruption if $\nu > \bar{\nu}$ and an informal policy with low corruption to one with high corruption if $\nu \leq \underline{\nu}$.*

If $\eta < 1$ and $\phi \leq \frac{1}{\mu w_2 f(w_1)}$ the politician can never implement an informal policy with low corruption, so any informal policy is one with high corruption.

Proof of Lemma 14. The proof follows the logic of the proof of Lemma 3. First note that the claim that, given assumption 2, $\tau_2^*(r=1) = \tau_2^*(r=0) = \Psi$ is independent of η so continues to hold, as does Lemma 6.

CASE 1: When $\phi \leq \frac{1}{\mu w_2 f(w_1)}$, $\tau_2 = f^{-1}\left(\frac{1}{\phi \mu w_2}\right) - w_1 \leq 0$, so there is no value of τ for which the honest bureaucrat takes additional bribes and the informal policy with high corruption can never happen. This proves the last statement of the Lemma.

CASE 2: When $\frac{1}{\mu w_2 f(w_1)} < \phi < \frac{1}{\mu w_2 f(\Psi)}$, the politician's objective function for $\tau \in [0, \tau_2]$ is:

$$V(\tau) = \nu \left[\mu F\left(f^{-1}\left(\frac{1}{\phi \mu w_2}\right)\right)\left(\lambda + \phi\lambda(1-\mu)\right) - \eta c^{-1}(1, D) - \tau\right] \\ + (1-\nu)\left[\mu F(\tau + e_H^*(\tau))\left(\lambda + \phi\lambda(1-\mu)\right) - \eta(e_H^*(\tau) - w_1) - \tau\right] + \mu\lambda - \Psi$$

And its derivative with respect to τ for $\tau \in [0, \tau_2]$ is:

$$\frac{\partial V(\tau)}{\partial \tau} = \nu(-1) + (1-\nu)\left[\mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1\right]$$

where $U_2 := \lambda + \phi\lambda(1-\mu)$. This derivative is positive if and only if:

$$\begin{aligned} &\mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1 \\ &\quad \geq \nu\left(\mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau}\right) \end{aligned} \tag{18}$$

However, unlike in the proof of Lemma 3, the left-hand side of inequality (18) is no longer necessarily positive. We now show that there exists some threshold $\tilde{\eta} \in [0, 1]$ such that $LHS(\eta) > 0 \Leftrightarrow \eta > \tilde{\eta}$. If $\eta = 1$, then the left-hand side of (18) is positive since $LHS(\eta) = \mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - 1 - \frac{\partial e_H^*(\tau)}{\partial \tau} > 0$ given that $1 + \frac{\partial e_H^*(\tau)}{\partial \tau} > 0$ (Lemma 6) and since $\mu U_2 f(\tau + e_H^*(\tau)) > 1$ as shown in the proof of Lemma 3. Instead, if $\eta = 0$, then the left-hand side of 18 reduces to: $LHS(\eta) = \mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - 1$. Suppose first that $LHS(0) = \mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - 1 \leq 0$. Since $\frac{\partial LHS(\eta)}{\partial \eta} = -\frac{\partial e_H^*(\tau)}{\partial \tau} > 0$, $\forall \eta \in [0, 1]$, we can apply the intermediate value theorem to conclude that there exists a unique $\tilde{\eta} \in [0, 1]$ such that $LHS(\eta) > 0 \Leftrightarrow \eta > \tilde{\eta}$. If instead, $LHS(0) = \mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - 1 > 0$, then we can define $\tilde{\eta} = 0$ and also state that $LHS(\eta) > 0 \Leftrightarrow \eta > \tilde{\eta}$.

We can now prove the first statements in the proof:

- Suppose first that $\eta < \tilde{\eta}$. In that case, condition (18) can never be satisfied since the left-hand side is negative while the right-hand side is positive. Therefore, $\frac{\partial V(\tau)}{\partial \tau} < 0$ for any $\tau \in [0, \tau_2]$. This implies that the optimal informal policy is an informal policy with high corruption.
- Suppose now that $\eta \geq \tilde{\eta}$. In that case, we can apply the arguments from the proof of Lemma 3 directly to show that $\frac{\partial V(\tau)}{\partial \tau} \geq 0$ if $\nu \geq \bar{\nu}$ and $\frac{\partial V(\tau)}{\partial \tau} \leq 0$ if $\nu \leq \underline{\nu}$.

CASE 3: When $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$, the derivative of the politician's objective function for $\tau \in [0, \tau_2]$ is:

$$\frac{\partial V(\tau)}{\partial \tau} = \begin{cases} \nu(-1) + (1-\nu)\left[\mu U_2 f(\tau + e_H^*(\tau))\left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1\right] \\ \qquad \text{if } \mu\phi w_2 f(\Psi) < c(\Psi - w_1, H) \text{ and } \tau < \tilde{\tau}, \\ -(1 - (1-\nu)\eta) \text{ otherwise.} \end{cases}$$

where $\tilde{\tau}$ is such that $\mu\phi w_2 f(\Psi) = c(\Psi - \tau - w_1, H)$ (see proof of Lemma 3).

Since $-(1-(1-\nu)\eta) < 0$ for any $\eta \in [0,1]$, then $\frac{\partial V(\tau)}{\partial \tau} < 0$ whenever $\mu\phi w_2 f(\Psi) \geq c(\Psi - w_1, H)$ or when $\mu\phi w_2 f(\Psi) < c(\Psi - w_1, H)$ and $\tau \geq \tilde{\tau}$. In these cases, an informal policy with high corruption is optimal.

When $\mu\phi w_2 f(\Psi) < c(\Psi - w_1, H)$ and $\tau < \tilde{\tau}$, $\frac{\partial V(\tau)}{\partial \tau}$ can be either positive or negative, depending on η and ν. Following the same logic as Case 2, there exists a threshold $\tilde{\eta}$ such that $\frac{\partial V(\tau)}{\partial \tau} < 0$ if $\eta < \tilde{\eta}$ while $\frac{\partial V(\tau)}{\partial \tau}$ can be either positive or negative when $\eta \geq \tilde{\eta}$. When $\eta \geq \tilde{\eta}$, we can define

$$\bar{\nu} = \max_{\tau \in [0,\tilde{\tau}]} \left\{ \frac{\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau}} \right\} \tag{19}$$

$$\underline{\nu} = \min_{\tau \in [0,\tilde{\tau}]} \left\{ \frac{\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1}{\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau}} \right\} \tag{20}$$

and $\frac{\partial V(\tau)}{\partial \tau} \geq 0$ if $\nu \leq \underline{\nu}$ and $\frac{\partial V(\tau)}{\partial \tau} \leq 0$ if $\nu \geq \bar{\nu}$.

In case 3, we can therefore define $\tilde{\eta} = 1$ if $\mu\phi w_2 f(\Psi) \geq c(\Psi - w_1, H)$ or if $\mu\phi w_2 f(\Psi) < c(\Psi - w_1, H)$ and $\tau \geq \tilde{\tau}$, and define $\tilde{\eta}$ as the solution to $\mu U_2 f(\tau + e_H^*(\tau)) \left(1 + \frac{\partial e_H^*(\tau)}{\partial \tau}\right) - \eta \frac{\partial e_H^*(\tau)}{\partial \tau} - 1$ when $\mu\phi w_2 f(\Psi) < c(\Psi - w_1, H)$ and $\tau < \tilde{\tau}$. Given these thresholds, the first statement of the Lemma also holds in this case. □

Next we compare the set of parameters for which the politician would prefer an informal policy with high corruption to an informal policy with low corruption.

Lemma 15. *The set of ν such that the politician prefers an informal policy with high corruption to one with low corruption for a given η, denoted $N(\eta)$, is larger when $\eta < 1$ than when $\eta > 1$: $N(\eta_H) \subseteq N(\eta_L)$, $\forall \eta_L < 1 \leq \eta_H$.*

Proof. For any ϕ, the set of ν such that the politician prefers an an informal policy with high corruption to one with low corruption can be written as $N(\eta) = \{\nu \mid \nu \geq \tilde{\nu}(\eta)\}$. When $\phi \leq \frac{1}{\mu w_2 f(w_1)}$ only an informal policy with low corruption is possible so $\tilde{\nu}(\eta) = 1$ for any η. When $\phi > \frac{1}{\mu w_2 f(w_1)}$, then $\tilde{\nu}(\eta)$ is either equal to 0, when $\eta < \tilde{\eta}$, or equal to $\bar{\nu}$ as defined in expression (1) or expression (19), when $\eta \geq \tilde{\eta}$. Finally, note that $\bar{\nu}$ is an increasing function of η (see proof of Lemma 3). Therefore, we can conclude that $\tilde{\nu}(\eta)$ is weakly increasing in η, so that $N(\eta_H) \subseteq N(\eta_L)$, $\forall \eta_H \geq \eta_L$, which proves the statement. □

A.7.2 High share of dishonest bureaucrats

We have shown that, when $\eta < 1$, the politician is weakly more likely to choose an informal policy with high corruption to one with low corruption when choosing among informal polices. We now show that, when she prefers an informal policy with high corruption over one with low corruption, the politician is also more likely to choose an informal policy than a formal policy when $\eta < 1$ compared to when $\eta > 1$.

We first note that Proposition 1 remains unchanged, but that the threshold $\bar{\phi}_H$ above which the politician chooses an informal policy with high corruption is now lower. For all the cases in which the politician always prefers an informal policy with high corruption to one with low corruption described in Lemma 14, we define $\bar{\nu} = 0$.

Proposition 10. *Suppose that $\eta < 1$ and that $\nu \geq \bar{\nu}$, then if the share of high-ability bureaucrats is sufficiently high, $\mu > \bar{\mu}_H$, there exists a unique threshold $\bar{\phi}_{H,\eta<1} \in [0,1)$ on the observability of public services such that the politician chooses an informal policy with high corruption if and only if $\phi > \bar{\phi}_{H,\eta<1}$. The threshold $\bar{\phi}_{H,\eta<1}$ is smaller than the threshold $\bar{\phi}_H$ defined in Proposition 1.*

Proof. First note that Part 1 in the proof of Proposition 1 does not rely on the assumption that $\eta \geq 1$, but a lower value of ϕ affects the value of $V(0)$. When $\phi \geq \frac{1}{\mu w_2 f(\Psi)}$, $V(0)$ becomes even larger when $\eta < 1$ so an informal policy with high corruption remains optimal. When $\phi \in \left[\frac{1}{\mu w_2 f(w_1)}, \frac{1}{\mu w_2 f(\Psi)}\right]$, a lower value of η affects the threshold under which condition (5) holds, and therefore the threshold on ϕ above which the politician chooses to implement an informal fiscal policy.

Second, note that the only argument in Part 2 that depends on η is Claim 2. Indeed, the existence of a $\bar{\mu}$ such that inequality (7) is satisfied if $\mu > \bar{\mu}$ requires η to be small enough: $\eta < \bar{\eta}$. However, since $\bar{\eta} > 1$ (see proof of Proposition 1), then $\eta < \bar{\eta}$ for any $\eta < 1$. So the threshold $\bar{\mu}$ exists for any $\eta < 1$.

Finally, recall that the threshold $\bar{\phi}_H$ is implicitly defined by the condition (as per Condition 6):

$$\Psi + \mu U_2 \left[\nu \left(F\left(f^{-1}\left(\frac{1}{\bar{\phi}_H \mu w_2}\right)\right) - 1\right) + (1-\nu)\left(F(e_H^*(0)) - 1\right)\right] - (1-\nu)\eta(e_H^*(0) - w_1) = 0 \quad (21)$$

Note that the left-hand side of equation (21) is decreasing in η as $e_H^* > w_1$ and increasing in ϕ if $\mu > \bar{\mu}$. As a result, the left-hand side of equation (21) is higher when $\eta < 1$ than when $\eta \geq 1$ so the value of ϕ such that equation (21) is satisfied must be lower when $\eta < 1$

than when $\eta \geq 1$. This proves that the threshold $\bar{\phi}_{H,\eta<1}$ is smaller than the threshold $\bar{\phi}_H$ defined in Proposition 1. □

A.7.3 Low share of dishonest bureaucrats

Finally, we can show that, in cases where the politician prefers an informal policy with low corruption to one with high corruption, the threshold $\bar{\phi}_L$ remains unchanged when $\eta < 1$.

Proposition 11. *Suppose that $\eta < 1$ and that $\nu \leq \bar{\nu}$, then if the share of high-ability bureaucrats is sufficiently high, $\mu > \bar{\mu}_H$, there exists a unique threshold $\bar{\phi}_{L,\eta<1} \in [0, 1)$ on the observability of public services such that the politician chooses an informal policy with low corruption if and only $\phi > \bar{\phi}_{L,\eta<1}$. The threshold $\bar{\phi}_{L,\eta<1}$ is the same as the threshold $\bar{\phi}_L$ defined in Proposition 2.*

Proof. This result follows from the facts that (1) the proof of Proposition 2 does not rely on the assumption that $\eta \geq 1$, and (2) neither inequality (8) nor inequality (9) depend on the value of η. □

A.8 Politician not held responsible for corruption

We consider a model in which voters only hold the politician accountable for the bribes they pay with some probability α. In this case, the politician's objective function becomes:

$$V(\tau) = \alpha \times \Bigg(\mathbb{E}_{\omega,\theta}\Big[\lambda F(\omega(\tau + e^*_\theta(\tau))) - \tau - \eta b^*_\theta(\tau) + \phi F(\omega(\tau + e^*_\theta(\tau)))\tilde{V}(r = 1) + (1 - \phi F(\omega(\tau + e^*_\theta(\tau))))\tilde{V}(r = 0)\Big]\Bigg) + (1 - \alpha) \times \Bigg(\mathbb{E}_{\omega,\theta}\Big[\lambda F(\omega(\tau + e^*_\theta(\tau))) - \tau + \phi F(\omega(\tau + e^*_\theta(\tau)))\tilde{V}(r = 1) + (1 - \phi F(\omega(\tau + e^*_\theta(\tau))))\tilde{V}(r = 1)\Big]\Bigg)$$

Where $\tilde{V}(r = 1) = \lambda F(\tau^*_2(r = 1)) - \tau^*_2(r = 1)$ and $\tilde{V}(r = 0) = \mu\lambda F(\tau^*_2(r = 0)) - \tau^*_2(r = 0)$. This simplifies to:

$$\mathbb{E}_{\omega,\theta}\Big[\lambda F(\omega(\tau + e^*_\theta(\tau))) - \tau - \alpha\eta b^*_\theta(\tau) + \phi F(\omega(\tau + e^*_\theta(\tau)))\tilde{V}(r = 1) + (1 - \phi F(\omega(\tau + e^*_\theta(\tau))))\tilde{V}(r = 0)\Big]$$

The politician's objective function is identical to the one in our baseline model, except for the fact that the bribe term $b^*_\theta(\tau)$ is now multiplied by $\alpha\eta$ instead of just η, where $\alpha < 1$. If

$\alpha\eta > 1$, then all the results from the baseline model carry over to this alternative model. If $\alpha\eta < 1$, which includes the case where the voters never hold the politician accountable, $\alpha = 0$, then Propositions 10 and 11 apply instead.

Appendix: For online publication

A.9 Appendix Tables

Table A1: Funding gap for police patrolling in India

Monthly Petrol Accounting

	(1)	(2)	(3)	(4)	(5)
VARIABLES	N	mean	sd	min	max
Average Budget	107	627.1	868.4	0	2,083
Vehicle Liters Petrol	169	174.5	79.87	0	567.4
Vehicle Petrol Expenditure	169	13,257	6,069	0	43,115
Vehicle Budget Balance	102	-12,440	5,837	-30,180	0
Motorcycle Liters Petrol	175	31.13	28.30	0	266.7
Motorcycle Petrol Expenditure	175	2,366	2,150	0	20,264
Motorcycle Budget Balance	105	-1,621	1,721	-8,132	2,083
Combined Budget Balance	101	-14,845	6,526	-33,858	-4,685

Authors' calculation from survey data. Estimates assume petrol prices of 75.99 INR per liter, the minimum daily price in Madhya Pradesh during November, 2018. Vehicle fuel mileage estimated at dealer-reported figure of 14.1 kilometers per liter for Tata Safari Storme. Motorcycle fuel mileage estimated at 60 kilometers per liter. Missing budget figures are due to non-reporting during survey interviews.

Table A2: Funding gap for police patrolling in India (treating missing values as zeros)

Monthly Petrol Accounting

	(1)	(2)	(3)	(4)	(5)
VARIABLES	N	mean	sd	min	max
Average Budget	180	372.8	736.2	0	2,083
Vehicle Liters Petrol	169	174.5	79.87	0	567.4
Vehicle Petrol Expenditure	169	13,257	6,069	0	43,115
Vehicle Budget Balance	169	-12,860	6,147	-43,115	0
Motorcycle Liters Petrol	175	31.13	28.30	0	266.7
Motorcycle Petrol Expenditure	175	2,366	2,150	0	20,264
Motorcycle Budget Balance	175	-1,982	2,255	-20,264	2,083
Combined Budget Balance	167	-15,256	7,004	-53,247	-3,422

Authors' calculation from survey data. Estimates assume petrol prices of 75.99 INR per liter, the minimum daily price in Madhya Pradesh during November, 2018. Vehicle fuel mileage estimated at dealer-reported figure of 14.1 kilometers per liter for Tata Safari Storme. Motorcycle fuel mileage estimated at 60 kilometers per liter. Missing budget figures are due to non-reporting during survey interviews and are counted as zero in this table.

Table A3: Citizen Survey: Are there bribes in this setting?

	Mean	N
How many times did you contact the department during the last year?		
1 to 5 times	0.71	1402
6 to 10 times	0.14	1402
11 to 20 times	0.04	1402
More than 20 times	0.01	1402
Never contacted	0.09	1402
To what extent do you face difficulties in contacting the department?		
To a great extent	0.19	1402
To quite an extent	0.43	1402
Can't say	0.18	1402
To a lesser extent	0.18	1402
Not at all	0.02	1402
What are the difficulties that are most faced while getting the services?		
No service provision without unofficial payments	0.65	1402
Unable to contact the concerned officials	0.55	1402
No clear information on the duration for these services	0.30	1402
Low quality of services	0.31	1402
Incorrect records	0.14	1402
Others	0.02	1402
Normally, what procedure do people adopt to get rid of the difficulties faced?		
Give a bribe	0.82	1402
Get undue favors through the politician	0.42	1402
Consult courts	0.41	1402
Lodge a complaint with the department	0.25	1402
Contact the provincial ombudsman	0.15	1402
Do nothing	0.04	1402
Disputes		
What normally are the reasons for disputes?		
Corruption in the system	0.51	1402
Influential people / land mafia	0.33	1402
Wrong distribution of land in the family	0.62	1402
No organized forum for land related issues	0.32	1402
Lack of education in the people	0.55	1402
What is the normal procedure that is adopted for the solution of these disputes?		
Unofficial means, bribes, and gifts	0.13	1400
Official legal procedure	0.20	1400
Through courts	0.23	1400
Through mutual understanding	0.10	1400
Through panchayat/politically or social investigation	0.20	1400
Through mutual consultation between elders of the families	0.13	1400
Do women and vulnerable groups face fraud and injustice?	0.62	1402