Friendly Lobbying under Time Pressure

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Abstract

Lobbyists often target legislators who are aligned with them rather than opponents. The choice of whom to lobby affects both what information becomes available to legislators and how much influence special interest groups exert on policies. However, the conditions under which aligned legislators are targeted are not well understood. We investigate how the pressure to conclude policies quickly affects the strategic decision of whom to lobby. We derive conditions on the cost of delaying policies and on the distribution of legislators' preferences for lobbyists to prefer targeting allies. We show that the use of allied intermediaries has important implications for the duration of policy making and the quality of policies. Counter-intuitively, an increase in time pressure can increase the duration of policy making and a longer duration does not always lead to better-informed policies.

Keywords: Dynamic Lobbying, Friendly Lobbying, Persuasion, Strategic Information Transmission

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1 Introduction

A growing empirical literature documents that lobbyists often target legislators who already support their preferred policies.[1] One of the reasons lobbyists target these 'friends' is that they can serve as intermediaries to persuade other legislators (Schnakenberg 2017; Awad 2020). Strategically sharing information with selected allies impacts both lobbyists' influence on policies and what information is available to legislators. The decision of whom to lobby can be a significant source of political influence and thus it is vital to better understand its determinants.

We focus on an under-explored determinant of lobbying strategies: the time pressure faced by legislators and lobbyists. When information on a policy is not immediately available, lobbyists need to incentivize legislators to wait for new information before voting. Lobbyists must therefore promise to provide sufficiently precise information in the future to persuade impatient legislators to delay the vote. We show that, as a consequence, time pressure restricts the set of intermediaries that lobbyists can rely on. When waiting is too costly, lobbyists no longer benefit from lobbying legislators privately but lobby publicly instead.

Delaying a vote to obtain more information is a common concern for policy makers. However, legislators often disagree on the value of waiting for that information. For example, when the decision to grant a permanent license to the ride-sharing operator Uber was delayed by two months, London Assembly Members had diverging views on whether this would generate valuable information. While some supported the extension to obtain additional data from the company, the Chair of the Assembly's transport committee argued that the delays would not generate valuable information: "What will [Transport for London] learn in two more months that it didn't learn in the last 15 months?" (Mathewson 2019).

In this case, the assembly members expected the information from the company to eventually be publicly available. In other cases, individual legislators are able to delay a vote because they expect to receive information from lobbyists privately. When a vote on the Biden administration's $3.5tn spending bill was delayed by a group of moderate Democrats in September 2021, one of them suggested the bill was moving too fast and that more information was needed: "Instead of rushing to spend trillions on new government programs and additional stimulus funding, Congress should hit a strategic pause on the budget-reconciliation legislation. [...] We must allow for a complete reporting and analysis of the implications a multitrillion-dollar bill will have for this generation and the next" (Manchin 2021). The media was quick to note that the legislators pushing to delay the bill, which included a

[1] Ainsworth (1997); Kollman (1997); Hojnacki and Kimball (1998, 1999); Baumgartner and Mahoney (2002); Hall and Miler (2008); Igan and Mishra (2014); Miller (2022).

reform of the drug pricing legislation, were some of the main recipients of funding from the pharmaceutical industry. They had regular direct interactions with the industry's lobbyists who could provide necessary information (Sirota and Perez 2021).

Both legislators and lobbyists are often under pressure to act quickly. In the spending bill example, one Democrat Representative pointed out that "the urgency is important, [...] We want to have it happen as soon as possible" (Kroll 2021). Chalmers (2013) witnessed the same urgency among lobbyists in the European Union: "Lobbyists [...] explain that there is an important premium on providing timely information in the EU. Information that is too late loses all of its value." The importance of timing in lobbying has been recently highlighted in empirical studies of dynamic lobbying (You 2017, 2020; Kim, Stuckatz and Wolters 2020). Yet, most theories of informational lobbying are either static or do not consider that delaying policies can impose costs on both legislators and lobbyists.

We propose a dynamic model of informational lobbying to address two main questions. First, how does time pressure affect lobbyists' choices of intermediaries and their preferences for private versus public lobbying? Second, how does the use of intermediaries affect the duration and quality of policy making?

In the model, a legislature decides between two policies. A lobbyist, who prefers one of the two policies, chooses how long to look for information and selects a legislator with whom to privately share that information. Information becomes more precise over time but waiting imposes a cost on the lobbyist and legislators. Legislators continuously choose whether to vote on the policy or to wait longer. Upon observing the lobbyist's verifiable information, the targeted legislator can share an unverifiable policy recommendation (an *endorsement*) with other legislators, thus acting as an intermediary between the lobbyist and the legislature. Other legislators draw inferences based on the targeted intermediary's endorsement and the time it took to obtain information. They form beliefs about the benefits of either policy, decide whether to hold a vote, and for which policy to vote.

In equilibrium, a majority of legislators either votes to stop the process immediately or waits until the lobbyist provides information. The lobbyist chooses a length of investigation and an intermediary such that the median legislator is exactly indifferent between stopping the process immediately and waiting for the lobbyist's information. The lobbyist faces a trade-off between choosing a more friendly intermediary and waiting longer. An intermediary who is more friendly to the lobbyist is more likely to be persuaded by the lobbyist's information, but makes a less persuasive recommendation from the median's perspective. The median thus requires more precise information to wait, forcing the lobbyist to run a longer investigation.

This trade-off determines the equilibrium duration and choice of intermediary.

Our first result is that an increase in time pressure always induces the lobbyist to select an intermediary who is more aligned with the median. A more aligned intermediary's endorsement is more valuable to the median and makes waiting more beneficial. As time pressure increases, the lobbyist needs to compensate the median for waiting by selecting an increasingly moderate intermediary.

Our second result is that, when time pressure becomes sufficiently high, the lobbyist no longer uses an intermediary. Inducing the median legislator to wait for an endorsement becomes too costly, so the lobbyist provides the information directly to her. This suggests that private lobbying should only happen on policies where time pressure is not too strong.

Our third result relates the duration of policy making to the cost of waiting. As time pressure increases, one would expect policies to conclude faster. However, because the lobbyist needs to incentivize the median to wait, he needs to promise sufficiently precise information. As time pressure increases, the lobbyist can therefore be forced to run a longer investigation, leading to a longer policy process.

Finally, we analyze how the quality of policy making depends on time pressure. Absent lobbying, greater time pressure would induce legislators to rush the process and generate less informed policies. In the presence of lobbying, waiting costs have an ambiguous effect on policy quality. Higher waiting costs force the lobbyist to choose a more moderate intermediary and thus generate more precise information for the median legislator. On the other hand, higher waiting costs can decrease the duration of the investigation and reduce the information's accuracy. We find that, for sufficiently small or sufficiently large waiting costs, time pressure increases the quality of policy making.

Our model provides a framework to interpret empirical patterns of lobbying strategies and legislative behavior. When legislators face significant time pressure, lobbyists are forced to target more moderate allies. This is consistent with evidence suggesting that targeting allied legislators is a less valuable lobbying strategy for highly salient policies (Baumgartner and Mahoney 2002) or policies that are subject to more constituency pressure (Hall and Miler 2008). Although legislators tend to act faster on more salient issues (Spendzharova and Versluis 2013), salience and constituency pressure do not always correspond to time pressure. Our results reveal that time pressure is an important determinant of lobbying strategies which is worth studying empirically. Political actors are under time pressure when delaying policies is costly, which can occur for a number of reasons. One reason delaying policies can be costly is that both legislators and lobbyists dislike the status quo. The longer they spend investigating the value of alternative policies, the longer they must endure the status quo, and the more pressure they face to replace it quickly. The source of pressure could also be different for legislators and lobbyists. Legislators might be under pressure because they face a crisis to resolve (such as responding to the COVID-19 pandemic) or because other interest groups are pressing them to quickly change the status quo (such as grassroots climate activists pressing for action on climate change). Lobbyists themselves can feel pressure to act quickly if they fear that legislators will obtain information internally or from a competing interest group. We study several extensions of our model to analyze these alternative sources of time pressure.

2 Related Literature

Our paper relates to the interest group literature and in particular to studies of the determinants of legislator targeting with informational lobbying.[2] Several theories of lobbying assume that an interest group can only *publicly* provide information to a collective body. These theories mainly focus on how information transmission is shaped by the preferences of the legislature (Schnakenberg 2015; Alonso and Câmara 2016). A key takeaway is that interest groups have more influence the more aligned policy makers are to the lobbyist and the more disagreement there is among policy makers. In other models, interest groups can privately provide information to legislators (Bardhi and Guo 2018; Chan et al. 2019). Caillaud and Tirole (2007) study how the private provision of information can help achieve the interest group's goals by letting some legislators observe information, and subsequently rely on them to persuade their peers.

Within the lobbying literature, several papers have proposed to rationalize the empirical regularity that lobbyists frequently interact with aligned legislators. Austen-Smith and Wright (1994) suggest that lobbyists do this to counteract persuasion by competing interest groups. Hall and Deardorff (2006) argue that it is easier for lobbyists to help allies exert effort towards achieving a shared policy objective. Groll and Prummer (2016) show that both the ideological preferences of legislators and their position in a network affect whether they are targeted by lobbyists. In Ellis and Groll (2020), lobbyists' preferences for targeting allies depend on legislators' resource constraints. Schnakenberg and Turner (2021) show that, due to signaling effects, lobbyists contribute to the campaigns of allies when donations are sufficiently likely to affect the likelihood of winning the election. Schnakenberg (2017) shows that when access is costly, lobbyists send a cheap talk message to friendly legislators who then persuade a majority. Awad (2020) studies a model of cheap talk and verifiable evidence

[2]There is also a literature on legislator targeting with other forms of lobbying, e.g., Chen and Zápal (2022); Judd (2022).

and focuses on the choice of allied intermediaries to improve presentation of information from the lobbyist's perspective. Minaudier (2022) shows that, in a static setting, lobbyists are more likely to lobby friends when legislators have access to internal information.

Unlike ours, these papers do not consider that information may take time to arrive. We show that this can restrict the possibility of targeting friends and is therefore crucial. Bennedsen and Feldmann (2002) and Dellis (2021) do allow the lobbyist to search for information as in our model, but this search is only costly for the lobbyist, not for the legislators. We show that, when legislators also bear this cost, lobbyists face additional constraints on the strategies they can choose.

From a technical perspective, our paper also relates to the dynamic persuasion literature. The idea that an information provider needs to generate sufficient information to induce the receiver to wait has been explored in Che, Kim and Mierendorff (2021). Our paper studies a similar question with multiple receivers but restricts the sender to providing concealable information through an intermediary rather than allowing him to choose more general information structures.

3 Model

We consider a lobbyist L and a continuum of legislators indexed by $i \in [0,1]$. Time is continuous and the game ends at some time, normalized to 1: $t \in [0,1]$. Legislators need to choose between two policies $x \in \{0,1\}$. The relative value of each policy depends on a state of nature, ω, distributed uniformly between 0 and 1: $\omega \sim U[0,1]$ and constant over time. The lobbyist can receive a signal s about ω, whose precision increases over time. That is, the longer the lobbyist waits before obtaining the signal, the more likely it is that it indicates the true state. If the signal is obtained at some time $t^* \in [0,1]$, the signal is equal to the true state, $s = \omega$ with probability t^* and with probability $1 - t^*$ the signal is uninformative $s = \tilde{s} \sim U[0,1]$, uncorrelated with the true state.

Actions. The lobbyist moves first and publicly chooses two actions at time $t = 0$. First, he chooses how long to carry out some research on ω. We denote by $\ell \in [0,1]$ the length of time he decides to investigate. In addition, he chooses one legislator, $j \in [0,1]$, as an intermediary and commits to transmitting information to only that legislator. We also allow the lobbyist to commit to share information publicly, which we denote $j = \emptyset$.

After the lobbyist moves, each legislator observes the lobbyist's choice of ℓ and j and votes at every instant $t \in [0,1]$ on whether to delay the policy choice or to hold a vote on the policy. We call this first vote a *procedural vote* and denote $p_{it} \in \{0,1\}$ legislator's i procedural vote at time t, with $p_{it} = 1$ denoting voting to stop. If a majority votes to stop, then the legislature holds a vote on the policy. Each legislator then votes for either option $x = 0$ or $x = 1$. Let $x_i \in \{0, 1\}$ denotes legislator i's *policy vote*. The option that receives a majority of votes is then implemented.

Before the lobbyist's investigation ends ($t < \ell$), players have no information about ω. At $t = \ell$, the lobbyist observes the signal realization s and chooses whether to disclose it to the legislator j that he selected as an intermediary. The signal s is hard evidence so the lobbyist cannot lie about it, but can withhold it. Let $\hat{s} \in \{s, \emptyset\}$ be the evidence reported by the lobbyist to the intermediary. Legislator j observes $\hat{s}$, updates her beliefs about ω, and then sends a message to endorse either $x = 0$ or $x = 1$. Let $m_j \in \{0, 1\}$ be the endorsement shared by legislator j. The intermediary's endorsement is cheap talk and not verifiable.

Still at $t = \ell$, other legislators observe legislator j's endorsement, before voting on the procedural vote and on a policy vote, if it is held. If a policy vote is held at any time t, the chosen policy is then implemented, the game ends, and payoffs are realized. If no policy vote has been held at any $t < 1$, the legislature holds a policy vote at $t = 1$.

To summarize, the timing is as follows:

1. Nature draws ω from the uniform distribution over $[0, 1]$.
2. The lobbyist publicly chooses a length of investigation $\ell \in [0, 1]$ and an intermediary $j \in [0, 1] \cup \{\emptyset\}$.
3. At every $t \in [0, 1]$, the legislators hold a procedural vote. If a majority agrees to hold a policy vote, the policy vote is held and a policy is chosen.
4. At time ℓ, if a policy vote has not been held before, the lobbyist observes s, and shares $\hat{s} \in \{s, \emptyset\}$ with legislator j.
5. Legislator j observes $\hat{s}$, and publicly endorses $m \in \{0, 1\}$.
6. Every other legislator $i \neq j$ observes m but not $\hat{s}$, chooses whether to hold a vote on the policy $p_i \in \{0, 1\}$ and if a majority votes to stop, votes for policy $x \in \{0, 1\}$.

Preferences. Each legislator i is identified by a parameter $\hat{x}_i \in [0, 1]$ distributed according to some distribution with full support on the interval $[0, 1]$. The median legislator's preference parameter is denoted $\hat{x}_M > \frac{1}{2}$. Legislator i's payoff from policy $x \in \{0, 1\}$ is given by:

$$u_i(x, \omega) = \begin{cases} 1 & \text{if } x = 1 \text{ and } \omega \geq \hat{x}_i, \\ 1 & \text{if } x = 0 \text{ and } \omega < \hat{x}_i, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Therefore, legislator i prefers policy $x = 1$ if the state is at least $\hat{x}_i$ and policy $x = 0$ otherwise. By contrast, the lobbyist prefers policy $x = 1$ independently of the state: his payoff is $v(x) = x$.

In addition, both the legislators and the lobbyist bear a cost k proportional to the time spent before choosing the policy.[3] Therefore, if at time t, some legislator i and the lobbyist expect policy x to be chosen at time $t' > t$, then their expected utilities are:

$$U_{it}(x, t') = \mathbb{E}_\omega[u_i(x, \omega)] - k(t' - t) \quad \text{and} \quad V_t(x, t') = x - k(t' - t).$$

Equilibrium and strategies. We look for weak Perfect Bayesian Equilibria in pure strategies. This requires sequentially rational strategies and beliefs that satisfy Bayes rule wherever possible. To rule out unintuitive equilibria, we make several assumptions which are standard in the lobbying and bargaining literature.

1. *Sincere voting on the policy vote.* Each legislator votes for the policy that maximizes her expected utility given her beliefs. This rules out, for example, equilibria where legislators always vote for the same policy because none of them are pivotal.

2. *As-if pivotal voting on procedural votes.* Legislators vote as if they are pivotal on the current and all future procedural votes. Legislators anticipate the outcome of any current or future policy vote given point 1. above and optimally choose when to hold this vote.

3. *Sincere endorsements.* When an intermediary is selected to make an endorsement, she does so sincerely. That is, the intermediary makes endorsement $m = 0$ if she prefers policy $x = 0$ and endorsement $m = 1$ otherwise.[4]

A formal definition of our equilibrium concept is provided in the Appendix (p. 1).

[3]The assumption that the lobbyist and legislators face the same cost k is effectively a normalization of the lobbyist's benefit from policy $x = 1$ as noted in Che, Kim and Mierendorff (2021).

[4]Assuming sincere endorsements is not necessary for the strategy profile we characterize to be an equilibrium. However, babbling equilibria are also possible if we do not assume that intermediaries are restricted to sincere endorsements.

Discussion of assumptions

Commitment. We assume that the lobbyist can commit both to a choice of intermediary and to a length of investigation before information is generated. These assumptions are often consistent with the behavior of lobbyists. For example, lobbyists gain access to legislators through campaign donations (see e.g., Kalla and Broockman 2016; Fouirnaies and Hall 2018) before policies are tabled, thus committing to interact with specific legislators.[5] Moreover, lobbyists often commission reports or surveys from lawyers or consultancy firms or might request data from their clients (Chalmers 2011). This requires them to set a deadline to obtain the information and prevents them from learning new information until then. In other cases, these assumptions might not be realistic. We discuss how they affect our results in Section 6.

Legislators' preferences. The legislators' preferences have a stark structure: they prefer one policy over the other as soon as the underlying state is above a threshold. Above that threshold, a higher state does not make their preferences for that policy stronger. In this sense, their preferences are similar to those assumed in case-base models of judicial politics (see e.g., Lax (2011)). However, these preferences can still be interpreted as spatial preferences where legislators with a lower threshold are more aligned with the lobbyist, and those with a higher threshold are on the opposite side of the ideological spectrum.

Time-independent state of nature. We assume that the state remains constant over time. In the real world, the state of nature can change over time in some policy areas. If the values of the state are sufficiently correlated over time, allowing the state to change would simply dampen the incentives to investigate for both players. If instead the state changes drastically, the information provided by the lobbyist might become irrelevant by the time the decision is made. This would effectively increase the cost of delaying the end of the investigation and would be factored into the cost k. In the extreme, if the state is constantly changing, the legislators would need to update the policy constantly. Our results apply to policies that are difficult to reverse and whose consequences can be predicted to some degree.

Other means of information acquisition. In practice, waiting longer is not the only way for lobbyists to increase the precision of information. Lobbyists can also use monetary resources to acquire information. In the Appendix (p. 21), we show that the model can be

[5] We do not allow the lobbyist to target several legislators. This is without loss of generality as Awad (2020) shows that lobbyists cannot gain from targeting multiple legislators when preferences are 'nested'.

extended to allow the lobbyist to invest resources to accelerate information acquisition. As long as the marginal cost of doing so is not too large, the lobbyist would invest resources which would allow him to choose a friendlier intermediary. However, our main results remain unchanged.

Intermediary's communication. We restrict the intermediary to make a binary cheap talk recommendation through her endorsement. Upon seeing some evidence $\hat{s}$, the intermediary is strictly in favor of either policy 1 or policy 0. As a result, the intermediary could not gain from using a larger set of messages. We also show in the Appendix (p. 22) that if the intermediary could share the hard evidence she obtained from the lobbyist, she would prefer not to share it or would share it in a way that is outcome-equivalent to the binary endorsement.[6]

4 Equilibrium Behavior

Our objective is to understand how time pressure affects the lobbyist's strategy. As a first step, it is helpful to understand how the legislators would structure information acquisition themselves. We begin by characterizing the duration that the median legislator would set if she were an agenda setter. The second step is to derive the duration that the lobbyist would choose if he were constrained to publicly sharing information. Finally, we characterize equilibrium strategies when the lobbyist can choose both the duration of the investigation and the intermediary.

4.1 Legislature's Preferred Duration

If the median chooses the investigation's duration, she trades off more precise information with the cost of delaying policy making. The precision of information affects legislators differently depending on their preferences. A legislator with threshold $\hat{x}_i > \frac{1}{2}$ chooses policy $x = 0$ by default since it is more likely that the state is below her threshold than above it and vice versa for Instead, a legislator with threshold $\hat{x}_i \leq \frac{1}{2}$ chooses policy $x = 1$ by default.

When the legislators in favor of policy $x = 0$ observe a signal s, they switch to policy $x = 1$ if they observe a sufficiently high signal ($s \geq \hat{x}_i$) and the information is sufficiently precise ($\ell \geq \frac{2\hat{x}_i - 1}{2\hat{x}_i}$). Similarly, a legislator in favor of policy $x = 1$ switches to supporting policy $x = 0$ if she sees a signal below her threshold that is sufficiently precise ($\ell > \frac{1-2\hat{x}_i}{2(1-\hat{x}_i)}$). Figure 1 illustrates the set of signals and lengths of investigation for which legislators switch their

[6] However, relaxing this assumption would allow for other equilibria, including ones in which the intermediary is forced to disclose all the lobbyist's evidence and the lobbyist no longer gains from private lobbying.

policy choice. Crucially, without a sufficiently long investigation, a legislator does not switch her vote.

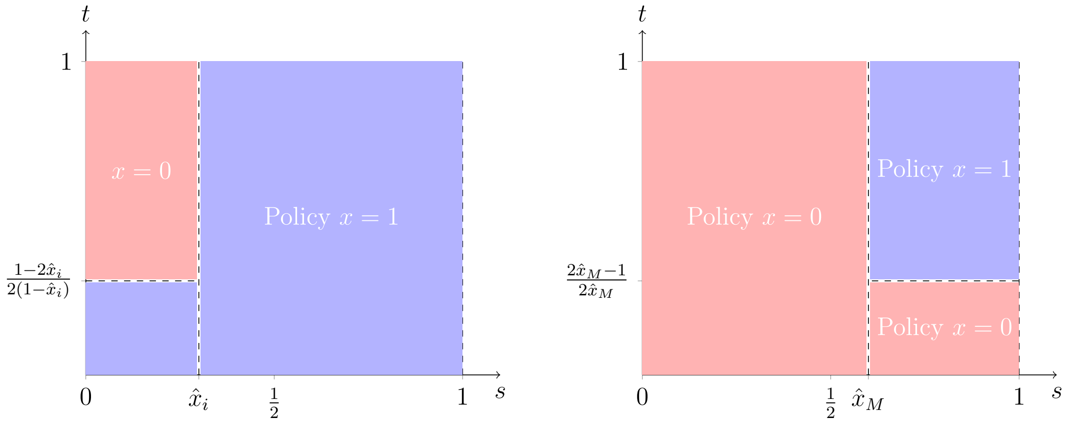

Figure 1: Policy choices as a function of signal s and precision t

Anticipating this voting behavior, the median chooses the duration ℓ to maximize the probability of making the correct policy decision net of the waiting cost. The median knows that for any $\ell \in \left[0, \frac{2\hat{x}_M-1}{2\hat{x}_M}\right)$, she (and therefore a majority of legislators) would still vote for policy $x = 0$ independently of the signal s, so her expected utility is strictly decreasing in that region. For $\ell \in \left[\frac{2\hat{x}_M-1}{2\hat{x}_M}, 1\right]$, her expected utility is linear in ℓ and increasing as long as the cost of waiting (k) is not too large. As a result, the optimal length of investigation is either $\ell_M^* = 0$ or $\ell_M^* = 1$. Waiting till the deadline is optimal if the marginal gain of waiting is above the marginal cost, k, and if the net expected utility at $\ell = 1$ is larger than the expected utility at $\ell = 0$.

Remark 1. *In equilibrium, the majority's optimal policy duration is $\ell_M^* = 1$ if the cost of waiting is sufficiently low: $k < 1 - \hat{x}_M$, and $\ell_M^* = 0$ otherwise.*

4.2 Public Disclosure

If the lobbyist must provide information publicly, he faces two constraints. First, when the information arrives, it must persuade the median to support policy $x = 1$. Second, he needs to persuade the median to wait long enough for the information to arrive.[7]

The lobbyist chooses the duration of the investigation to maximize the probability that the median chooses policy $x = 1$, net of the cost of waiting. When information is generated, the lobbyist needs to ensure that the information is sufficiently precise that the median is persuaded by a favorable signal ($s \geq \hat{x}_M$).

[7]We show in the Appendix (p. 2) that there is always a majority of legislators that supports the median's votes on the procedural and policy decisions so we can focus on the median's behavior.

In addition, the lobbyist faces a constraint at the start of the game. He needs to provide sufficiently precise information that the median legislator is willing to wait. Since the median legislator expects policy $x = 0$ to be chosen if the process stops immediately, her expected payoff equals the probability that the state is below her cutoff: $\mathbb{P}(\omega < \hat{x}_M) = \hat{x}_M$. Therefore, the lobbyist needs to promise to wait long enough that the median legislator's expected utility at ℓ^* is at least $\hat{x}_M$.

When these two constraints are satisfied, the probability of persuading the median is simply the probability that the signal s exceeds the threshold $\hat{x}_M$, which is equal to $1 - \hat{x}_M$ and is independent of the duration. Therefore, as soon as a duration ℓ satisfies both constraints, the lobbyist has no reason to investigate longer because waiting is costly. When waiting is so costly that the median is never willing to wait, the process stops immediately.

Remark 2. *When the lobbyist publicly provides information, the optimal ℓ^* is given by:*

1. $\ell^*_P = \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k}$ *if* $k \leq 1 - \hat{x}_M$.
2. $\ell^*_P = 0$ *if* $k > 1 - \hat{x}_M$.

When time pressure is not too high, the median would run the longest possible investigation if she could choose the duration herself. Instead, when the lobbyist chooses the duration, he chooses the shortest duration such that the median is willing to wait and the endorsement is persuasive.

4.3 Selective Disclosure through Intermediaries

We now analyze the case where the lobbyist can choose both the duration of the investigation and an intermediary with whom to share the information. We begin by deriving the set of intermediaries and duration such that the median follows the intermediary's endorsement and is willing to wait for that endorsement. We then solve for the optimal duration and choice of intermediary for the lobbyist within this set.

4.3.1 Persuading the Legislature to Support the Lobbyist's Policy

For a given duration of investigation, the set of intermediaries that can help the lobbyist achieve his preferred policy is determined by two requirements. First, the intermediary needs to find the information sufficiently precise that she chooses (and endorses) policy $x = 1$ when the signal is high enough and policy $x = 0$ otherwise. Second, the intermediary needs to have preferences sufficiently similar to those of the median that the median follows the intermediary's endorsement. In other words, the intermediary herself needs to be persuadable, and her subsequent endorsement must be persuasive.

Intermediary's endorsements. Suppose the lobbyist has access to legislator j with threshold $\hat{x}_j$. For the intermediary's endorsement to be informative, it is necessary that she sometimes prefers policy $x = 0$ and sometimes policy $x = 1$ upon observing evidence $\hat{s} \in [0, 1]$. If for instance she preferred policy $x = 1$ no matter the signal $\hat{s}$ she observed, she would always make the endorsement that leads the legislature to choose policy $x = 1$ and information transmission would break down. Upon observing a signal s, legislator j prefers policy $x = 1$ if the signal is sufficiently high and precise. She therefore gives endorsement $m_j = 1$ when she observes evidence above her threshold $s \geq \hat{x}_j$ and the investigation lasted sufficiently long. Otherwise, she gives endorsement $m_j = 0$.[8]

Following the intermediary's endorsement. Suppose that the intermediary gives an endorsement $m_j = 1$ if and only if $s \geq \hat{x}_j$ and the investigation lasted sufficiently long. What is the set of intermediaries j and duration ℓ that leads a majority to follow the intermediary's endorsement? Since all legislators other than the intermediary observe the same information, the median is decisive. Therefore, we focus on the inferences drawn by the median legislator and her decision following an endorsement. Upon endorsement $m_j = 1$, the median infers that the signal belongs to the interval $[\hat{x}_j, 1]$, so the probability that the state is above $\hat{x}_M$ is $\mathbb{P}(\omega \geq \hat{x}_M | s > \hat{x}_j)$. The median prefers policy $x = 1$ if that probability is above $\frac{1}{2}$. This is the case provided that the intermediary's threshold is not too far from the median's threshold and the information is sufficiently precise. Endorsement $m_j = 1$ from an intermediary with threshold $\hat{x}_j \leq \hat{x}_M$ after an investigation of length ℓ persuades the median legislator to vote for $x = 1$ if

$$\ell \geq \frac{(2\hat{x}_M - 1)(1 - \hat{x}_j)}{2(1 - \hat{x}_M)\hat{x}_j}. \tag{2}$$

If this condition does not hold, the median and all legislators to her right vote for policy $x = 0$ for any endorsement. Finally, if the legislator's threshold is less than that of the median, $\hat{x}_j \leq \hat{x}_M$, the median legislator always prefers policy $x = 0$ when the intermediary gives endorsement $m_j = 0$.

An intermediary with a threshold larger than that of the median ($\hat{x}_j > \hat{x}_M$) can also make endorsements that persuade the median. Since the intermediary is harder to persuade than the median, the median always follows the intermediary's endorsement in favor of policy $x = 1$. The necessary condition to persuade the median is therefore that the intermediary is

[8]Given this strategy, the lobbyist knows that the intermediary will not endorse $x = 1$ when she observes evidence $s < \hat{x}_j$. In that case, the lobbyist prefers to conceal his evidence. Therefore, the lobbyist's strategy is simply to disclose the signal, $\hat{s} = s$, if $s \geq \hat{x}_j$ and to conceal it, $\hat{s} = \emptyset$, if $s < \hat{x}_j$.

persuaded to support policy $x = 1$: $\ell \geq \frac{2\hat{x}_j - 1}{2\hat{x}_j}$.

Figure 2 illustrates the set of intermediaries $\hat{x}_j$ and lengths of investigation ℓ that satisfy these conditions. As the duration increases, the set of intermediaries that can be used by the lobbyist (the blue area) expands. A longer duration means that the information is sufficiently precise that an intermediary with a threshold far below that of the median is persuasive and an intermediary with a threshold far above that of the median is persuadable.

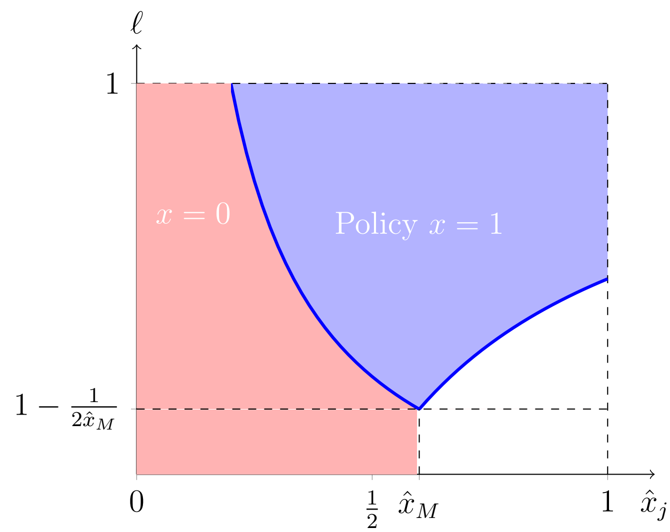

Figure 2: Set of persuasive intermediaries and duration

Using intermediaries has two effects. First, it increases the likelihood that the lobbyist finds persuasive evidence. When the lobbyist must disclose information publicly, he only obtains his preferred policy when his investigation generates evidence $s \in [\hat{x}_M, 1]$. When the lobbyist can use an intermediary with threshold $\hat{x}_j < \hat{x}_M$, he increases the set of persuasive signal realizations to $[\hat{x}_j, 1] \supset [\hat{x}_M, 1]$. Second, using an intermediary garbles the information available to the other legislators. This makes them more likely to choose the wrong policy and therefore decreases the value of information. For instance, the median would choose policy $x = 1$ following $s \in [\hat{x}_j, \hat{x}_M)$ with an intermediary, but not when s is public.

4.3.2 Persuading the Legislature to Wait

The lobbyist needs to choose ℓ and j such that a majority is willing to wait until the end of the investigation. The duration of the investigation affects the total waiting cost that the legislators expect to face and therefore their decision whether to start the process at all. The choice of intermediary affects the value of the information that is generated at the end of the process and therefore the benefit of waiting for that information.

Each legislator faces the following choice. If the investigation stops immediately, they anticipate that a majority of legislators would vote for policy $x = 0$, since the state is more likely to be below the median's threshold than above it ($\hat{x}_M > \frac{1}{2}$). Legislator i's expected payoff from stopping immediately is therefore the probability that the state is below her threshold, which equals $\hat{x}_i$. Stopping at any point between time $t = 0$ and the time at which the lobbyist is expected to share information with the intermediary is worse than stopping immediately, as the legislature would still vote for policy $x = 0$, but the legislators would bear the cost of waiting.

At the end of the investigation, when the lobbyist shares information, the legislators expect two possible scenarios: Either the lobbyist has observed evidence above the intermediary's threshold $s \geq \hat{x}_j$ and the intermediary endorses policy $x = 1$, or $s < \hat{x}_j$ and the intermediary endorses policy $x = 0$. Since the legislators do not expect any additional information to arrive after that point, all legislators would vote to stop the process.

Hence, at time $t = 0$, legislators anticipate that if they do not stop the process immediately, it will continue until the end of the investigation ℓ^*, at which point policy $x = 1$ will be chosen with probability $\mathbb{P}(s \geq \hat{x}_j)$ and policy $x = 0$ with probability $\mathbb{P}(s < \hat{x}_j)$. In addition, all legislators to the left of the median anticipate that they will prefer policy $x = 1$ when it is chosen, and all legislators to the right of the median anticipate that they will prefer policy $x = 0$ when it is chosen. Therefore, it is sufficient to persuade the median to wait until time $t = \ell^*$ for a majority of legislators to vote to continue at every point until ℓ^*.

The median votes to continue the process until time $t = \ell^*$ if the utility she expects to get from the information at ℓ^* net of the cost of waiting is greater than the expected utility she expects to get if policy $x = 0$ is chosen immediately. She is therefore willing to wait if:

$$\mathbb{P}(m_j = 1)\,\mathbb{P}(\omega \geq \hat{x}_M | m_j = 1) + \mathbb{P}(m_j = 0)\,\mathbb{P}(\omega < \hat{x}_M | m_j = 0) - k\ell \geq \hat{x}_M.$$

This condition requires a sufficiently long investigation given the choice of intermediary:

$$\ell \geq \frac{(2\hat{x}_M - 1)(1 - \hat{x}_j)}{2(1 - \hat{x}_M)\hat{x}_j - k}. \tag{3}$$

4.3.3 Optimal Lobbying Strategies

For a given choice of intermediary, the probability of persuading the median to choose policy $x = 1$ is independent of the duration as long as the duration is above the minimum threshold defined by inequality (2). Similarly, persuading the legislature to wait does not depend on duration as long as inequality (3) is satisfied. However, increasing the duration allows the lobbyist to choose an intermediary with a lower threshold who is easier to persuade. A longer duration, however, decreases the lobbyist's utility through the cost of waiting. The lobbyist therefore faces a trade-off between selecting an intermediary who is easier to persuade and running a shorter investigation. All else equal, the lobbyist would prefer a lower duration and an intermediary with a lower threshold. However, the lobbyist also needs to promise sufficiently precise information to ensure that legislators wait. The lobbyist can achieve this by either increasing the duration or choosing a more moderate intermediary.

Our first main result is to characterize the choice of intermediary and duration that optimally resolves this trade-off for the lobbyist. We use the following definitions to characterize equilibrium strategies.

Definition 1. *An investigation is thorough if $\ell = 1$ and is rushed otherwise.*

Definition 2. *Lobbying is private if the lobbyist shares the information privately with a selected intermediary with threshold $\hat{x}_j \neq \hat{x}_M$. Lobbying is public if $\hat{x}_j = \hat{x}_M$.*

Proposition 1 below summarizes the lobbyist's optimal choice. Figure 3 depicts the equilibrium lobbying strategy as a function of the cost of waiting and the median's threshold and Figure 4 illustrates the equilibrium duration in these strategies.

Proposition 1. *Given a median ideal point $\hat{x}_M \in \left(\frac{1}{2}, 1\right)$, there exist three thresholds $k_1(\hat{x}_M) < k_2(\hat{x}_M) < k_3(\hat{x}_M)$, such that, in equilibrium, the lobbyist chooses ℓ^* and $\hat{x}_j^*$ as follows:*

1. *If $k \leq k_1(\hat{x}_M)$, the lobbyist runs a thorough investigation and engages in private lobbying.*
2. *If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$, the lobbyist runs a rushed investigation and engages in private lobbying.*
3. *If $k \in [k_2(\hat{x}_M), k_3(\hat{x}_M)]$, the lobbyist runs a rushed investigation and engages in public lobbying.*
4. *If $k > k_3(\hat{x}_M)$, the lobbyist does not lobby ($\ell^* = 0$).*

Proposition 1 identifies four cases depending on the level of time pressure. When waiting is not very costly, the lobbyist prefers a longer duration to select a more friendly intermediary. As a result, the optimal duration of the investigation is the maximum possible duration ($\ell^* = 1$) which allows the lobbyist to choose an intermediary with a low threshold.

When the cost is higher but not too high, the lobbyist finds it too costly to wait until the deadline and rushes the investigation. This forces him to choose a more moderate intermediary. As the cost becomes high, the lobbyist runs out of intermediaries to use and switches to public lobbying. Using intermediaries would require such a long duration to

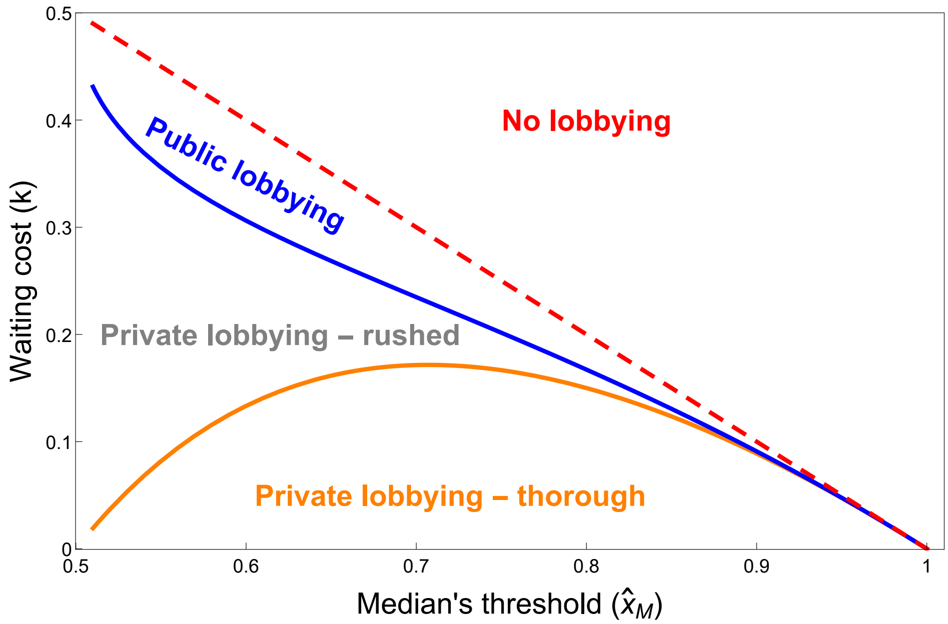

Figure 3: Equilibrium lobbying strategies.

induce the median to wait that the lobbyist is happy to send the information directly to the median or, equivalently, to share it publicly.

Finally, if the cost is too high, the lobbyist cannot persuade her to wait and gives up on persuasion.

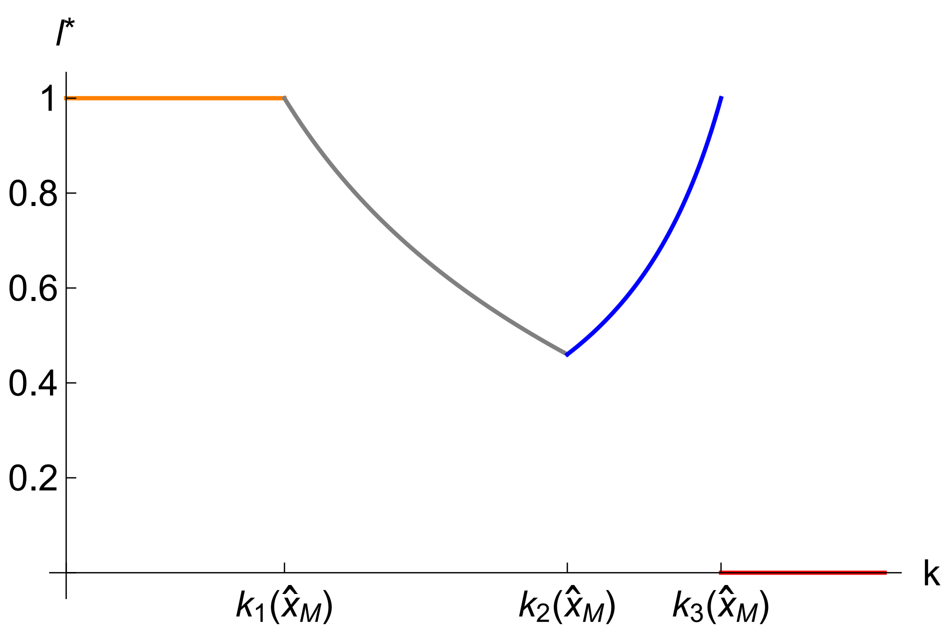

Figure 4: Equilibrium duration as a function of waiting costs. Figure drawn for $\hat{x}_M = 0.6$.

5 Time Pressure, Intermediaries, and Duration

Proposition 1 establishes that the lobbyist uses different strategies when the cost of waiting and the median voter's threshold change. In this section, we look at how time pressure affects the choice of intermediary, the equilibrium duration, and how well-informed policies are. Our main result is that higher time pressure always induces the lobbyist to choose a more moderate intermediary. However, we also show that higher time pressure can lead to both longer or shorter equilibrium duration.

5.1 Choice of Intermediary

To illustrate the role of time pressure, consider first a situation without waiting costs ($k = 0$). In this case, the lobbyist no longer needs to persuade the legislators to wait. He is still constrained to choose a sufficiently large duration because a message that is not sufficiently precise will not persuade the intermediary to ever change her preferred policy. He is also constrained to choose a sufficiently moderate intermediary whose endorsement persuades the median. The optimal strategy in this case is a special case of Proposition 1.

Corollary 1. *Without waiting costs ($k = 0$), the lobbyist selects intermediary $\hat{x}_j^* = 2\hat{x}_M - 1$ and the longest feasible investigation $l^* = 1$.*

With $k > 0$, the median is no longer willing to wait for information if she does not expect that information to be sufficiently precise. Therefore, the lobbyist must promise more information, which can be achieved by sharing information with an intermediary closer to the median. We call such an intermediary more moderate. A friendly intermediary is one who would have chosen the lobbyist's preferred policy in the absence of information. Instead, the lobbyist engages in confrontational lobbying when he targets a legislator who needs additional information to be persuaded.

Definition 3. *An intermediary is more moderate if her threshold is closer to that of the median. Lobbying is friendly if the lobbyist targets an intermediary with a threshold below $\frac{1}{2}$ and is confrontational if he targets an intermediary with a threshold above $\frac{1}{2}$.*

Proposition 2 shows that the more pressing a policy matter is, the more aligned the intermediary will be with the median, as illustrated in Figure 5.

Proposition 2. *As the waiting cost $k \in (0, k_3(\hat{x}_M))$ increases, the intermediary becomes weakly more moderate. When waiting costs are low, the lobbyist engages in friendly lobbying. For intermediate waiting costs, he engages in confrontational lobbying. For sufficiently large waiting costs he engages in public lobbying.*

As time pressure increases, the median requires more surplus to be willing to wait. The

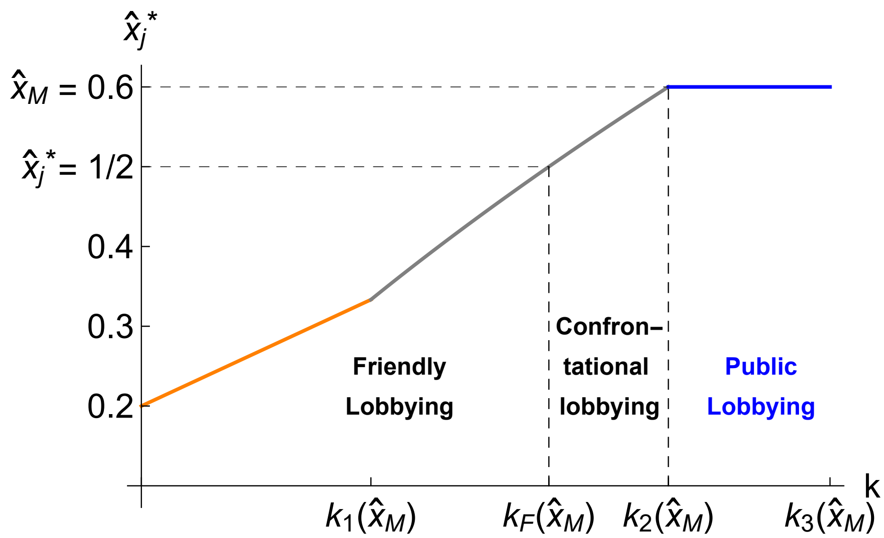

Figure 5: Equilibrium intermediary as a function of waiting costs. Figure drawn for $\hat{x}_M = 0.6$.

lobbyist can achieve that in two ways. Either by increasing the duration of the investigation or by choosing a more moderate intermediary. Increasing the duration imposes a cost on both the lobbyist and the legislator. Therefore, the marginal cost of increasing the duration to persuade the legislators to wait is higher than that of choosing a more moderate intermediary.

5.2 Duration

The model also generates predictions about the duration of policy making. When waiting costs are not too high, the equilibrium duration is positive. When costs are sufficiently low ($k \leq k_1(\hat{x}_M)$), the lobbyist finds it optimal to wait until the deadline ($\ell^* = 1$) so the equilibrium duration corresponds to the duration that would have been chosen by the median. The equilibrium duration also corresponds to that chosen by the median when the costs are very high, because the median is never willing to wait. However, for intermediate costs, the lobbyist's investigation is rushed. A majority prefers to obtain more information but the lobbyist only promises just enough information to ensure the median waits.

This conflict of interest between the lobbyist and the legislators generates a non-monotonic effect on equilibrium duration. In particular, the duration may increase as time pressure increases. Proposition 3 summarizes the effect of time pressure on duration. This effect is illustrated in Figure 4 above.

Proposition 3. *If the equilibrium duration of policy making ℓ^* is either* 0 *or* 1, *then it is independent of the waiting cost k. Otherwise, as the waiting cost k increases, the equilibrium duration of policy making ℓ^* initially decreases in k and then increases in k.*

Intuitively, the increase in duration occurs because the legislature needs to be persuaded to wait. The lobbyist would like to stop as early as possible to avoid additional waiting costs, but if the investigation is not long enough, a majority would stop immediately and choose policy $x = 0$. As waiting costs increase, the surplus that needs to be promised to the legislators becomes higher, so the lobbyist must investigate longer to make information more precise.

The decreasing region is due to the lobbyist's trade-off when choosing whether to target a more moderate intermediary or to run a longer investigation. In that region, the marginal cost of a more moderate intermediary is lower than the marginal cost of a longer investigation. Hence, the lobbyist runs a shorter investigation and persuades the median to wait by committing to target a more moderate legislator. Eventually, the lobbyist can no longer persuade the median to wait by targeting a more moderate intermediary, and therefore runs a longer investigation.

5.3 Implications for Policy Making

The model also generates implications about how well-informed policies are. If information takes time to generate, one would expect that the longer a policy takes to conclude, the more information is available, and the less likely the median is to choose the wrong policy. As a result, the more patient legislators are, the higher the quality of policy making should be. However, we show that when information is generated by lobbyists this is not necessarily true. A longer duration can correspond to less well-informed policies, and higher waiting costs can lead to better-informed policies.

We measure the quality of a policy, how well-informed it is, as the probability that it is the *correct policy* for a majority. Policy $x = 1$ is the *correct policy* for legislator i if the state exceeds her threshold: $\omega \geq \hat{x}_i$, while policy $x = 0$ is the correct one otherwise. Let $F(k, \hat{x}_M)$ denote this probability. Proposition 4 summarizes the non-monotonic relationship between time pressure and the quality of policy, illustrated in Figure 6.

Proposition 4. *When waiting costs are sufficiently low ($k \leq k_1(\hat{x}_M)$)) or sufficiently high ($k > k_2(\hat{x}_M)$)) the quality of policy is weakly increasing in the waiting cost. For intermediate waiting costs ($k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$), the quality of policy can be increasing or decreasing in the waiting cost, depending on the median's preferences.*

When the lobbyist provides information, the length of the investigation is not the only determinant of the quality of policy making. As information is provided through an intermediary, the median observes garbled information. If the intermediary and the median are less aligned, the median observes less precise information. Therefore, the quality of policy increases as the

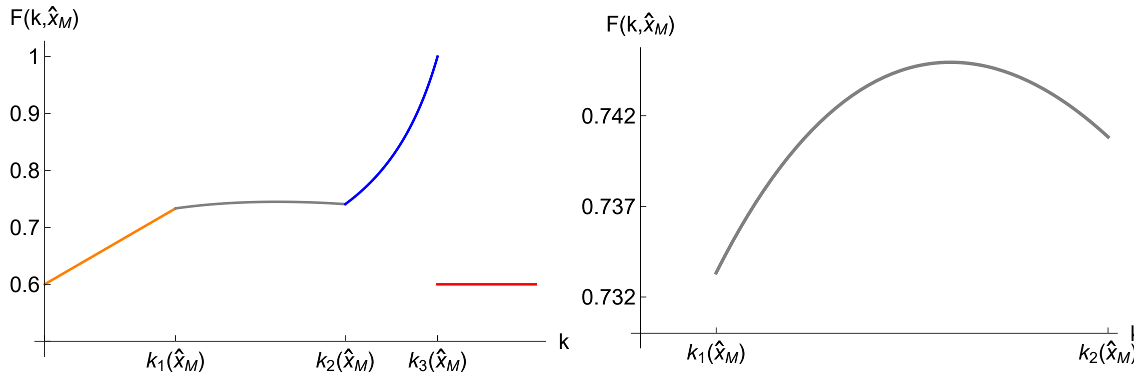

Figure 6: Quality of policy choice as a function of waiting costs. The right panel zooms in on the region of $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$. Figures drawn for $\hat{x}_M = 0.6$.

intermediary becomes more moderate. Since higher waiting costs force the lobbyist to select a more moderate intermediary, it can increase the amount of available information.

When higher waiting costs lead to a weakly longer policy duration (if $k \leq k_1(\hat{x}_M)$ or $k > k_2(\hat{x}_M)$), higher waiting costs unambiguously lead to better policies. When higher waiting costs lead to a shorter policy duration, however, the quality of policy can increase or decrease depending on which of the two effects—more moderate intermediary or shorter duration—dominates.

6 Extensions

We now discuss the robustness of our results to relaxing some of our assumptions. In particular, we analyze the case where the lobbyist cannot commit to a length of investigation or to an intermediary. We then explore two potential sources of endogenous time pressure: the possibility for legislators to obtain their own information and the interest group competition.

Commitment to duration. Lobbyists cannot always publicly commit to a length of investigation. Without commitment, the lobbyist would stop investigating as soon as he has acquired enough information to persuade the median. Anticipating this, the median would prefer to stop immediately. In the Appendix (p. 13), we show that, to solve this commitment problem, the lobbyist can seek access to a legislator who is *harder* to persuade than the median ($\hat{x}_j > \hat{x}_M$). Recall that any legislator needs an investigation that lasts at least $\ell \geq \frac{2\hat{x}_j - 1}{2\hat{x}_j}$ to find a signal $s \geq \hat{x}_j$ persuasive. Therefore, committing to intermediary $\hat{x}_j > \hat{x}_M$ prevents the lobbyist from deviating to a shorter investigation. If he did, he would always fail to generate a favorable endorsement from that intermediary. By choosing a sufficiently extreme intermediary, the lobbyist convinces a majority that he will generate enough information to make it worth their wait. Without commitment, the lobbyist can therefore still benefit from private lobbying and time pressure still increases both duration and policy quality. However, the intermediary now becomes more extreme as time pressure increases.[9]

Commitment to intermediary. Lobbyists are not always able to commit to a given intermediary at the start of the legislative process. Without commitment, the lobbyist could, in principle, choose a different intermediary once he observes the evidence. Whether such a deviation is profitable depends on the beliefs that the legislators form about the signal observed by the lobbyist upon hearing an unexpected endorsement. We show in the Appendix (p. 14) that the equilibrium characterized in Proposition 1 remains an equilibrium in this modified game. In this game, the legislators can form beliefs that lead a majority of them to vote in favor of policy $x = 0$ whenever they observe an endorsement made by a different legislator than the one with whom they expected the lobbyist to share information. As a result, it is unprofitable for the lobbyist to deviate to sharing evidence with any other legislator. While it co-exists with other equilibria, the equilibrium we characterized is the lobbyist-preferred equilibrium, and is payoff-equivalent to any other equilibria for the median legislator.

Pressure from legislators' internal information. One reason lobbyists feel under pressure to share information early is that legislators could be running a parallel internal investigation. In the Appendix (p. 17), we analyze how this affects the lobbyist's strategy. We assume that there is no cost of waiting but instead allow the legislators to run an investigation. The longer they investigate, the more likely they are to discover the state. Legislators can stop their investigation at any time before ℓ. The lobbyist's equilibrium strategy depends on how quickly the legislators can obtain internal information. In particular, we show that when they can obtain this information not too fast nor too slowly, the lobbyist shortens his investigation and chooses a more moderate intermediary. In this case, the legislators are likely to learn the state from their own information. The lobbyist therefore needs to move earlier to reduce the chances that the legislators discover the true state and ensure that they use his information instead. As a result, the legislature's ability to acquire information creates an endogenous form of time pressure which also leads the lobbyist to run a shorter investigation and target a more moderate intermediary. Another implication is that higher

[9] This commitment problem would also arise if the lobbyist could observe and disclose information before the end of the investigation (at some $t < \ell^*$). A potential benefit of observing early information would be for the lobbyist to stop a hopeless investigation. The lobbyist would only gain from obtaining early information if that benefit outweighs the cost from losing commitment power.

time pressure does not necessarily generate more incentives for the legislature to acquire its own information since acquiring internal information affects the information provided by the lobbyist.[10]

Pressure from competing interest groups. One source of pressure for lobbyists is the presence of competing interest groups. A competing lobbyist could provide information early on to induce the legislature to stop the process and prevent the other lobbyist from sharing its own biased information. We show in the Appendix (p. 20) that, with two opposed lobbyists who face no time pressure, (1) if a lobbyist chooses the same duration and intermediary as without competition, the competing lobby would have an incentive to preempt this information provision and (2) both lobbyists choosing the maximum duration and targeting the median legislator is always an equilibrium. Like time pressure, competition forces the lobbyist to choose a more moderate intermediary. Just like time pressure in the main model, however, competitive pressure does not necessarily lead to a shorter process.

Alternatively, competing interest groups not represented by professional lobbyists may put pressure on the legislators to act quickly. Consider a group of citizens who prefer policy $x = 0$ but do not have the resources to collect information like the lobbyist in our model. Instead, this group might be able to pressure the legislators by organizing grassroots activities such as protests or media campaigns until the government acts. In our model, this would correspond to increasing k. Since in equilibrium a higher k forces the lobbyist to choose a more moderate intermediary, it increases the probability that policy $x = 0$ is enacted. Therefore, competing interest groups with no resources to lobby the government would have incentives to endogenously generate this sense of urgency. However, note that if the interest group does not prefer policy $x = 0$ over policy $x = 1$ but simply wants the status quo changed as soon as possible, our model suggests that increasing the pressure k on legislators to act quickly could backfire, since it can lead to a longer duration in equilibrium.

7 Empirical Implications

Implications for lobbying studies. Scholars have mostly focused on two policy dimensions: the ideological preferences of legislators and the need for expertise. This is the case in studies measuring lobbying returns (de Figueiredo and Silverman 2006; Richter, Samphantharak and Timmons 2009; Goldstein and You 2017), lobbying connections and revolving doors (Blanes I Vidal, Draca and Fons-Rosen 2012; Bertrand, Bombardini and Trebbi 2014; Shepherd and You 2020; Miller 2022), and campaign donations (Bombardini and Trebbi

[10] Minaudier (2022) shows that internal and external information can be either strategic complements or substitutes in a static setting, even without explicit costs of acquiring information.

2011; Kim, Stuckatz and Wolters 2020).

Our results show that time pressure is another important dimension of policies affecting lobbying decisions. Two policies facing the same distribution of legislator preferences and need for expertise can induce different lobbying strategies depending on the pressure to act quickly and the time it takes to obtain information. Proposition 2 generates testable predictions relating how pressing a policy matter is and the lobbying strategies that one should expect. Less pressing issues are more likely to involve private lobbying, while more pressing issues are more likely to involve open lobbying or lobbying of more moderate and pivotal legislators. Propositions 2 and 3 taken together also generate testable predictions about the type of information that different legislators receive. Counter-intuitively, a more friendly legislator — who is easier to persuade — receives more precise information. This occurs because of the subtle trade-offs involved when more precise information takes time to gather but delaying decisions is costly.

Measuring time pressure. There can be various sources of time pressure, which, in our model, corresponds to the cost of delaying policies. First, some policies might face inherent urgency. This is the case for crisis legislation such as during the COVID-19 pandemic or the Michigan water contamination crisis. Both the lobbyist and the legislators agree that it is better to resolve the crises sooner rather than later, but disagree on how to resolve it. Second, some policies might not be inherently urgent but both the lobbyist and the legislators might dislike the status quo more than either version of a reform. The longer the delay in agreeing on a reform, the longer all parties have to endure the status quo. Finally, time pressure might arise endogenously because of interest group competition or because of the legislature acquires information internally, as we showed in Section 6.

Our model suggests that time pressure is an important determinant of lobbying strategies. When it is an exogenous feature of policies, it could be measured based on the ideological preferences of lobbyists and legislators for new policy proposals over the status quo. Other institutional features such as the number of key policies that legislators want to address in priority during their mandate could also proxy for time pressure. Measuring time pressure is more challenging when it is generated endogenously but our results show the importance of accounting for competing lobbies and the capacity of legislatures to generate internal information.

Finally, our model suggests that for a given level of urgency, the cost of delaying policy decisions should be assessed in relation to the speed of obtaining information. For a given level of time pressure, information that takes more time to arrive is effectively equivalent to increasing the cost of waiting. We should therefore expect relatively new and complex policy areas such as nanotechnologies or artificial intelligence to put legislators under more time pressure than issues where more information is already available, such as the health impact of smoking even if legislators faced the same urgency to resolve them.

8 Conclusion

This paper proposes a theory of informational lobbying in which lobbyists can use allied legislators as intermediaries but in which information acquisition takes time. We characterize the lobbyist's equilibrium strategy and generate testable predictions about the choice of intermediaries and policy duration.

Our results show that policies in which legislators face higher time pressure force the lobbyist to target more moderate intermediaries. However, more time pressure does not necessarily lead to more expedited policies. Both the time pressure faced by legislators and their policy preferences affect the long-run quality of policies. More patient legislators do not necessarily obtain more information as they allow the lobbyist to target more extreme intermediaries. More time pressure, rather than leading to rushed deliberations, can lead to better informed policies.

Time pressure is therefore an important consideration in the debate about the influence of special interest groups. Time pressure affects who lobbyists talk to, whose campaign lobbies donate to, and whom to hire when engaging in revolving-door lobbying.

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Appendix

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A Proofs of Propositions in the text

A.1 Equilibrium definition

An equilibrium consists of:

1. A pair (ℓ^*, j^*) for the lobbyist;
2. a disclosure strategy for the lobbyist: $\sigma : [0,1] \to [0,1] \cup \emptyset$ s.t. $\sigma(s) \in \{s, \emptyset\}$;
3. a procedural vote strategy $\pi_{it} : H_{it} \to \{0,1\}$ for all $i \in [0,1]$;
4. a policy vote strategy $\chi_{it} : H_{it} \to \{0,1\}$ for all $i \in [0,1]$;
5. belief functions over the state $f_{it}^{\omega} : H_{it} \to \Delta[0,1]$ and the evidence shown by the lobbyist to the intermediary j, $g_{it}^{\hat{s}} : H_{it} \to \Delta[0,1]$ for all $i \neq j$;
6. an endorsement strategy for the intermediary: $\mu_{it} : H_{it} \to \{0,1\}$ if $i = j^*$;
7. a belief function for the intermediary over the evidence observed by the lobbyist: $g_{it}^{s} : H_{it} \to \Delta[0,1]$ if $i = j^*$,

such that:

1. Legislators $i \neq j$ vote sincerely on the policy choice:

$$x_{it}(h_{it}) = 1 \Leftrightarrow \int_{\omega \in [0,1]} u_i(x=1,\omega) f_{\omega}(h_{it}) d\omega \geq \int_{\omega \in [0,1]} u_i(x=0,\omega) f_{\omega}(h_{it}) d\omega$$

2. Legislator $i = j$ sincerely recommends $m_j(h_{jt}) = 1$ following $\hat{s}$ if and only if:

$$\int_{\omega \in [0,1]} u_j(x=1,\omega) f_{\omega}(h_{jt}) d\omega \geq \int_{\omega \in [0,1]} u_i(x=0,\omega) f_{\omega}(h_{jt}) d\omega$$

   Given conditions 1. and 2., define $\tilde{x}(m_j)$ as the expected outcome of the policy vote and $\tilde{m}_{jt} = \begin{cases} m_j & \text{if } t \geq \ell^* \\ \emptyset & \text{if } t < \ell^* \end{cases}$. Finally, define $\tilde{\pi}_{it}$ as legislator i's expectation over future outcomes of the procedural vote if the process has not stopped yet, given i's own strategy and assuming that she is pivotal on future procedural votes. If the process has stopped at some $s < t$, let $\tilde{\pi}_{it} = 0$. Then, $\tilde{\pi}_{it} = \begin{cases} 1 & \text{if } \pi_{it}(h_t) = 1 \\ 0 & \text{if } \pi_{it}(h_t) = 0 \text{ or } \pi_{is} = 1 \text{ for some } s < t \end{cases}$.

3. Legislators vote as if pivotal on current and all future procedural votes: $\forall\ t \in [0,1]$,

$$p_{it} = 1 \Leftrightarrow \int_{\omega \in [0,1]} u_i(\tilde{x}(\tilde{m}_{jt}),\omega) f_{\omega}(h_{it}) d\omega \geq$$

$$\int_{\tau \in [t,1]} \sum_{\tilde{m}_{j\tau} \in \{0,1,\emptyset\}} \mathbb{P}(\tilde{m}_{j\tau}) \int_{\omega \in [0,1]} \mathbb{1}\{\tilde{\pi}_{i\tau}(\tilde{m}_{j\tau}) = 1\} \left[u_i(\tilde{x}(\tilde{m}_{j\tau}),\omega) - k(\tau - t)\right] f_{\omega}(h_{i\tau}, \tilde{m}_{j\tau}) d\omega d\tau$$

4. Beliefs are updated according to Bayes rule.

A.2 Proof of Remark 1

Consider a legislator's policy choice with threshold $\hat{x}_i$ and signal s:

$$x = \begin{cases} 1 & \text{if } \mathbb{P}(\omega \geq \hat{x}_i|s) \geq \mathbb{P}(\omega < \hat{x}_i|s) \\ 0 & \text{if } \mathbb{P}(\omega \geq \hat{x}_i|s) < \mathbb{P}(\omega < \hat{x}_i|s) \end{cases}$$

Therefore, legislator i chooses $x = 1$ iff $\mathbb{P}(\omega \geq \hat{x}_i|s) \geq \frac{1}{2}$. Using Bayes rule,

$$\mathbb{P}(\omega \geq \hat{x}_i|s) = \begin{cases} \ell \times 1 + (1-\ell) \times (1-\hat{x}_i) & \text{if } s \geq \hat{x}_i \\ \ell \times 0 + (1-\ell) \times (1-\hat{x}_i) & \text{if } s < \hat{x}_i \end{cases}$$

So, $\mathbb{P}(\omega \geq \hat{x}_i|s) \geq \frac{1}{2}$ if $s \geq \hat{x}_i$ and $\ell \geq \frac{2\hat{x}_i-1}{2\hat{x}_i}$ or if $s < \hat{x}_i$ and $\ell \leq \frac{1-2\hat{x}_i}{2(1-\hat{x}_i)}$.

Therefore, the median legislator expects the following policy choice outcomes:

1. If $\ell < \frac{2\hat{x}_M-1}{2\hat{x}_M}$, then $\tilde{x} = 0 \; \forall s$, since all legislators $\hat{x}_i \geq \hat{x}_M$ vote $x_i = 0$. Indeed, $\forall \hat{x}_i \geq \hat{x}_M$, if $s < \hat{x}_M$ then $s < \hat{x}_i$ and if $s \geq \hat{x}_M$, $\ell < \frac{2\hat{x}_i-1}{2\hat{x}_i}$.
2. If $\ell \geq \frac{2\hat{x}_M-1}{2\hat{x}_M}$, then $\tilde{x} = 0$ for $s < \hat{x}_M$, since all legislators $\hat{x}_i \geq \hat{x}_M$ vote $x_i = 0$, but $\tilde{x} = 1$ for $s \geq \hat{x}_M$, since all legislators $\hat{x}_i \leq \hat{x}_M$ vote $x_i = 1$.

Suppose that when the median proposes a duration ℓ, a majority of legislators vote to continue at every $t < \ell$ and vote to stop at every $t \geq \ell$. The median's expected utility at $t = 0$ is:

$$\begin{aligned} U_M(l) &= \begin{cases} \mathbb{P}(\omega \leq \hat{x}_M) - k\ell & \text{if } \ell < \frac{2\hat{x}_M-1}{2\hat{x}_M} \\ \mathbb{P}(s \geq \hat{x}_M)\,\mathbb{P}(\omega \geq \hat{x}_M|s \geq \hat{x}_M) + \mathbb{P}(s < \hat{x}_M)\,\mathbb{P}(\omega < \hat{x}_M|s < \hat{x}_M) - k\ell & \text{if } \ell \geq \frac{2\hat{x}_M-1}{2\hat{x}_M} \end{cases} \\ &= \begin{cases} \hat{x}_M & \text{if } \ell < \frac{2\hat{x}_M-1}{2\hat{x}_M} \\ \ell[2\hat{x}_M(1-\hat{x}_M) - k] + (1-\hat{x}_M)^2 + \hat{x}_M^2 & \text{if } \ell \geq \frac{2\hat{x}_M-1}{2\hat{x}_M} \end{cases} \end{aligned}$$

Therefore, if $2\hat{x}(1-\hat{x}_M) - k < 0$, then $U_M(\ell)$ is decreasing in ℓ for any $\ell \in [0,1]$. If $2\hat{x}(1-\hat{x}_M) - k > 0$, then $U_M(\ell)$ is decreasing in ℓ for $\ell \in [0, \frac{2\hat{x}_M-1}{2\hat{x}_M}]$ and increasing for $\ell \in [\frac{2\hat{x}_M-1}{2\hat{x}_M}, 1]$. Finally, we also need that the median's utility is higher at $\ell = 1$ than at $\ell = 0$:: $[2\hat{x}_M(1-\hat{x}_M) - k] + (1-\hat{x}_M)^2 + \hat{x}_M^2 \geq \hat{x}_M \Leftrightarrow 1 - \hat{x}_M \geq k$. Since $\hat{x}_M > \frac{1}{2}$, then $2\hat{x}_M(1-\hat{x}_M) > 1 - \hat{x}_M$, so $\ell = 1$ if $k \leq 1 - \hat{x}_M$.

When M proposes duration $\ell_M^* = 1$, the strategy $p_{it} = 0$ if $t < \ell_M^*$ and $p_{it} = 1$ if $t \geq \ell_M^*$ is optimal for a majority of legislators. Indeed, for any $\hat{x}_i \leq \hat{x}_M$, the expected policy utility from stopping at any $t < \ell_M^*$ is $\hat{x}_i$ and the expected policy utility from stopping at $t \geq \ell_M^*$ is

$$\begin{aligned} U_i(\ell_M^*) &= \mathbb{P}(s \geq \hat{x}_M)\,\mathbb{P}(\omega \geq \hat{x}_i|s \geq \hat{x}_M) + \mathbb{P}(s < \hat{x}_M)\,\mathbb{P}(\omega < \hat{x}_i|s < \hat{x}_M) \\ &= \ell_M^*[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i \end{aligned}$$

Therefore, at any $t \geq \ell_M^*$, all legislators $\hat{x}_i \leq \hat{x}_M$ prefer to stop since $\ell_M^*[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i > \ell_M^*[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i - kdt,\ \forall dt > 0$. For $t < \ell_M^*$,

1. Suppose by contradiction that some legislator $\hat{x}_i \leq \hat{x}_M$ has the following strategy: $p_{it} = 0$ for $t < \tau$ for some $\tau \in [0, \ell_M^*)$ and $p_{it} = 1$ for every $t \geq \tau$. This would require that: $\ell[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i - kdt < \hat{x}_i$ for any $dt > 0$. This requires that $\ell[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i - \hat{x}_i < 0$, that is $\ell[2\hat{x}_i(1-\hat{x}_M)] + (1-2\hat{x}_i)(1-\hat{x}_M) < 0$. This requires $\ell_M^* < \frac{2\hat{x}_i - 1}{2\hat{x}_i}$, which cannot hold as $\ell_M^* \geq \frac{2\hat{x}_M - 1}{2\hat{x}_M} > \frac{2\hat{x}_i - 1}{2\hat{x}_i}$. This also rules out the strategy $p_{it} = 1,\ \forall t \in [0, 1]$.

2. Suppose instead that some legislator $\hat{x}_i \leq \hat{x}_M$ has the following strategy, for some $\tau \in (0, \ell_M^*)$: $p_{it} = \begin{cases} 1 & \text{for } t < \tau \\ 0 & \text{for } t \in [\tau, \ell_M^*) \\ 1 & \text{for } t \geq \ell_M^*. \end{cases}$

   For this to be an equilibrium, it needs to be that at $t = \tau - dt$, $\hat{x}_i > u_i(\ell_M^*) - k(\ell_M^* - (\tau - dt))$ for any $dt > 0$. Otherwise, at that point the legislator would deviate to voting $p_{it} = 0$ instead of stopping. This requires:

   $$\begin{aligned} &\hat{x}_i > \ell_M^*[2\hat{x}_i(1-\hat{x}_M)] + (1-\hat{x}_i)(1-\hat{x}_M) + \hat{x}_M\hat{x}_i - k(\ell_M^* - (\tau - dt)) \\ &\Leftrightarrow 0 > \ell_M^*[2\hat{x}_i(1-\hat{x}_M) - k] + (1-2\hat{x}_i)(1-\hat{x}_M) + k(\tau - dt) \end{aligned}$$

   However, note that $\ell_M^*[2\hat{x}_M(1-\hat{x}_M) - k] + (1-\hat{x}_M)^2 + \hat{x}_M^2 \geq \hat{x}_M$ implies $\ell_M^*[2\hat{x}_i(1-\hat{x}_M) - k] + (1-2\hat{x}_i)(1-\hat{x}_M) \geq 0$ for any $\hat{x}_i \leq \hat{x}_M$, so for any $\tau \in (0, \ell_M^*)$ the inequality above cannot hold.

3. Points 1 and 2 rule out that legislators with $\hat{x}_i \leq \hat{x}_M$ ever stop just before ℓ_M^* or stop immediately at $t = 0$. These arguments can also be used to rule out any other strategies where the legislator stops for some period $[\tau_1, \tau_2] \subset [0, \ell_M^*]$.

When $\ell_M^* = 0$, there is no benefit for any legislator to ever vote $p_{it} = 0$ since the policy outcome never changes, so all legislators support the median's proposal.

Therefore, when the median can act as an agenda setter, the median offers either to stop immediately or to continue until $t = 1$ and a majority of legislators always vote on the procedural vote in such a way that the median's proposal is carried out.

A.3 Proof of Remark 2

If the lobbyist provides information publicly, his problem can be reduced to the optimal choice of duration as if he was providing the information to the median. Suppose that there always exists a majority of legislators supporting the decision of the median. Then the lobbyist's optimal strategy solves:

$$\begin{aligned} & \max_{\ell} && 1-\hat{x}_M-k\ell \\ \text{s.t} & \quad (1) && \ell \geq \frac{2\hat{x}_M-1}{2\hat{x}_M} \\ & \quad (2) && \ell \geq \frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{2\hat{x}_M(1-\hat{x}_M)-k} \end{aligned}$$

The first constraint ensures that the median is persuaded by a signal $s \geq \hat{x}_M$. The second constraint ensures that the median is willing to wait till ℓ rather than stop immediately. First note that constraint (2) implies constraint (1) since $\frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{2\hat{x}_M(1-\hat{x}_M)-k} \geq \frac{2\hat{x}_M-1}{2\hat{x}_M}$. Second note that the lobbyist's objective function is strictly decreasing in ℓ. Therefore, it is optimal to set constraint (2) binding: $\ell_D^* = \frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{2\hat{x}_M(1-\hat{x}_M)-k}$.

Next we show that, given this strategy, there indeed always is a majority of legislator supporting the median's decision. Note that $\frac{2\hat{x}_M-1}{2\hat{x}_M} \geq \frac{2\hat{x}_i-1}{2\hat{x}_i}$ for any $\hat{x}_i \leq \hat{x}_M$, so if constraint (1) is satisfied for $\hat{x}_M$, it is satisfied for all $\hat{x}_i \leq \hat{x}_M$. Similarly, $\ell \geq \frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{2\hat{x}_M(1-\hat{x}_M)-k}$ implies that $\ell \geq \frac{(2\hat{x}_i-1)(1-\hat{x}_M)}{2\hat{x}_i(1-\hat{x}_M)-k}$ for all $\hat{x}_i \leq \hat{x}_M$, so if constraint (2) is satisfied, then it is satisfied for all $\hat{x}_i \leq \hat{x}_M$. Therefore, if the lobbyists chooses ℓ_D^*, then:

1. A majority of legislators vote $p_{it} = 0$ iff $t < \ell_D^*$.
2. At $t = \ell_D^*$, a majority of legislators vote $x_i = 1$ if $s \geq \hat{x}_M$ and a majority of legislators vote $x_i = 0$ if $s < \hat{x}_M$.

Finally, for this to be an equilibrium, we also need to show that the lobbyist cannot get a majority that excludes the median to support a better proposal. First, given the policy choice of legislators derived in the proof of Remark 1, there is never a majority voting for $x = 1$ if $\ell < \frac{2\hat{x}_M-1}{2\hat{x}_M}$. Second, suppose that $\ell < \frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{2\hat{x}_M(1-\hat{x}_M)-k}$. In this case, for all $\hat{x}_i \geq \hat{x}_M$, $\ell < \frac{(2\hat{x}_i-1)(1-\hat{x}_M)}{2\hat{x}_i(1-\hat{x}_M)-k}$. So all these legislators would vote to stop the process immediately, which would constitute a majority.

A.4 Proof of Proposition 1

The proof proceeds as follows. We first derive the endorsement decisions of each possible intermediary and the associated beliefs and policy votes. We then derive the expected utility of the legislators for different combinations of duration and intermediary, and derive conditions for a majority to wait until the proposed duration. Finally, we derive the optimal duration and intermediary choice for the lobbyist given these constraints.

A.4.1 Intermediary endorsement

Suppose that the lobbyist sends the information to intermediary j. Consider an equilibrium strategy where the intermediary reports $m_j = 1$ if and only if she observes $s \geq \hat{x}_j$.

Given this strategy, the legislators form the following beliefs:

$$\mathbb{P}(\omega \geq \hat{x}_M | m_j = 1) = \mathbb{P}(\omega \geq \hat{x}_M | s \geq \hat{x}_j) = \begin{cases} \ell \times \frac{1-\hat{x}_M}{1-\hat{x}_j} + (1-\ell) \times (1-\hat{x}_M) & \text{if } \hat{x}_M \geq \hat{x}_j \\ \ell \times 1 + (1-\ell) \times (1-\hat{x}_M) & \text{if } \hat{x}_M < \hat{x}_j \end{cases}$$

$$\mathbb{P}(\omega < \hat{x}_M | m_j = 0) = \mathbb{P}(\omega < \hat{x}_M | s < \hat{x}_j) = \begin{cases} \ell \times 1 + (1-\ell) \times \hat{x}_M & \text{if } \hat{x}_M \geq \hat{x}_j \\ \ell \times \frac{\hat{x}_M}{\hat{x}_j} + (1-\ell) \times \hat{x}_M & \text{if } \hat{x}_M < \hat{x}_j \end{cases}$$

The median chooses $x = 1$ if $\mathbb{P}(\omega \geq \hat{x}_M | m_j) \geq \frac{1}{2}$. Following $m_j = 0$, we always have $\mathbb{P}(\omega \geq \hat{x}_M | m_j) < \frac{1}{2}$ in any informative equilibrium and for any $\ell \in [0, 1]$. Following $m_j = 1$ and given the intermediary's strategy, we have:

$$\mathbb{P}(\omega \geq \hat{x}_M | m_j = 1) \geq \frac{1}{2} \Leftrightarrow \begin{cases} \ell \geq \frac{(2\hat{x}_M - 1)(1-\hat{x}_j)}{2(1-\hat{x}_M)\hat{x}_j} & \text{if } \hat{x}_M \geq \hat{x}_j \\ \ell \geq \frac{2\hat{x}_M - 1}{2\hat{x}_M} & \text{if } \hat{x}_M < \hat{x}_j \end{cases}$$

Next, given the intermediary's strategy, we note that whenever the median is persuaded to vote for policy $x \in \{0, 1\}$, so is a majority since:

1. When the median votes $x_M = 1$, then:
   (a) Either $m_j = 1$, $\hat{x}_j > \hat{x}_M$, and $\ell \geq \frac{2\hat{x}_M - 1}{2\hat{x}_M}$. So $\hat{x}_j > \hat{x}_i$ and $\ell \geq \frac{2\hat{x}_i - 1}{2\hat{x}_i}$ for all $x_i \leq \hat{x}_M$ so that majority votes for $x = 1$.
   (b) Or $m_j = 1$, $\hat{x}_j \leq \hat{x}_M$ and $\ell \geq \frac{(2\hat{x}_M - 1)(1-\hat{x}_j)}{2(1-\hat{x}_M)\hat{x}_j}$, so $s \geq \hat{x}_i$ and $\ell \geq \frac{2\hat{x}_i - 1}{2\hat{x}_i}$ for all $\hat{x}_i \leq \hat{x}_j$. In addition, $\ell \geq \frac{(2\hat{x}_i - 1)(1-\hat{x}_j)}{2(1-\hat{x}_i)\hat{x}_j}$ for any $\hat{x}_i \in [\hat{x}_j, \hat{x}_M]$ since the right-hand side is increasing in $\hat{x}_i$. So all legislators such that $\hat{x}_i \leq \hat{x}_M$ vote for $x = 1$.
2. When the median votes $x_M = 0$, then:
   (a) Either $m_j = 0$, in which case all legislators believe $s < \hat{x}_j$, and all legislators such that $\hat{x}_i \geq \frac{1}{2}$ believe that $\mathbb{P}(\omega \geq \hat{x}_i | s < \hat{x}_j) < \frac{1}{2}$, for any $\hat{x}_j \in [0, 1]$.
   (b) Or $m_j = 1$ and $\hat{x}_j \leq \hat{x}_M$ but $\ell < \frac{(2\hat{x}_M - 1)(1-\hat{x}_j)}{2(1-\hat{x}_M)\hat{x}_j}$. In this case, all $\hat{x}_i$ such that $\ell < \frac{(2\hat{x}_i - 1)(1-\hat{x}_j)}{2(1-\hat{x}_i)\hat{x}_j}$ vote $x = 0$ and this constitutes a majority as it includes all the legislators with $\hat{x}_i \in [\hat{x}_M, 1]$.
   (c) Or m_j is uninformative, in which case legislators have the same belief as the prior and therefore all $\hat{x}_i > \frac{1}{2}$ also vote $x = 0$.

We can therefore define a set of legislators that would recommend policy $x = 1$ if and only if $s \geq \hat{x}_j$ and such that a majority would follow their endorsement.

Lemma 1. *For every $\hat{x}_M \in [\frac{1}{2}, 1]$ and for each duration $\ell \geq \frac{2\hat{x}_M - 1}{2\hat{x}_M}$, there exists a set of legislators $[\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$ such that:*

1. *Any legislator $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$ shares endorsement $m_j = 1$ if and only if $s \geq \hat{x}_j$*

2. *The legislature follows j's endorsement if* $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$

Proof of Lemma 1. Define $\underline{x}_j(\ell, \hat{x}_M) = \max\left\{\frac{1-2\ell}{2(1-\ell)}, \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}\right\}$. Note that $\hat{x}_j \geq \underline{x}_j(\ell, \hat{x}_M)$ then implies $\ell \geq \max\left\{\frac{1-2\hat{x}_j}{2(1-\hat{x}_j)}, \frac{(2\hat{x}_M-1)(1-\hat{x}_j)}{2(1-\hat{x}_M)\hat{x}_j}\right\}$ and that, for any $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \hat{x}_M]$, $\ell \geq \frac{(2\hat{x}_M-1)(1-\hat{x}_j)}{2(1-\hat{x}_M)\hat{x}_j} \Rightarrow \ell \geq \frac{(2\hat{x}_M-1)}{2\hat{x}_M} \geq \frac{(2\hat{x}_j-1)}{2\hat{x}_j}$. This implies that for any $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \hat{x}_M]$, (1) the intermediary shares $m_j = 1$ if and only if $s \geq \hat{x}_j$ and (2) the median and therefore a majority of legislators follow each endorsement.

Next, define $\bar{x}_j(\ell, \hat{x}_M) = \frac{1}{2(1-\ell)}$. Since $\frac{1}{2(1-\ell)} > \frac{1}{2}$, this upper bound only restricts thresholds $\hat{x}_j > \frac{1}{2}$. If $\hat{x}_j \geq \bar{x}_j(\ell, \hat{x}_M)$, then $\ell \geq 1 - \frac{1}{2\hat{x}_j}$, so all $\hat{x}_j \in [\hat{x}_M, \bar{x}_j(\ell, \hat{x}_M)]$ recommend $m_j = 1$ iff $s \geq \hat{x}_j$ and (2) the median and a majority of legislators follow each endorsement. □

A.4.2 Procedural vote

Given a proposed duration ℓ and an intermediary $\hat{x}_j$, we derive the set of legislators i for whom the following procedural vote strategy is an equilibrium strategy, given the expected outcome of the policy vote derived above. We refer to this strategy as 'waiting until ℓ:

$$p_{it} = \begin{cases} 0 & \forall t < \ell \\ 1 & \forall t \geq \ell \end{cases}$$

At any $t < \ell$, if legislator i stops the vote at that point, the legislator expects policy $x = 0$ to be chosen by the legislature, so expects to get $U_i(\tilde{x} = 0) = \mathbb{P}(\omega < \hat{x}_i) = \hat{x}_i$.

At $t = \ell$, if legislator i stops the vote, the legislator expects the following outcomes:

1. If $\hat{x}_j \notin [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$, then legislator i expects policy $x = 0$ to be chosen so expects to get $U_i(\tilde{x} = 0) = \hat{x}_i$.
2. If $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$, then legislator i expects policy $x = 1$ to be chosen with probability $\mathbb{P}(s \geq \hat{x}_j)$. So her expected utility is:

$$\begin{aligned} U_i(\ell, \hat{x}_j) &= \mathbb{P}(m_j = 1)\,\mathbb{P}(\omega \geq \hat{x}_i | m_j = 1) + \mathbb{P}(m_j = 0)\,\mathbb{P}(\omega < \hat{x}_i | m_j = 0) \\ &= \ell\left[\mathbb{P}(\omega \geq \hat{x}_j)\frac{\mathbb{P}(\omega \geq \hat{x}_i \cap \omega \geq \hat{x}_j)}{\mathbb{P}(\omega \geq \hat{x}_j)} + \mathbb{P}(\omega < \hat{x}_j)\frac{\mathbb{P}(\omega < \hat{x}_i \cap \omega < \hat{x}_j)}{\mathbb{P}(\omega < \hat{x}_j)}\right] \\ &+ (1-\ell)\left[\mathbb{P}(\tilde{s} \geq \hat{x}_j)\,\mathbb{P}(\omega \geq \hat{x}_M) + \mathbb{P}(\tilde{s} < \hat{x}_j)\,\mathbb{P}(\omega < \hat{x}_M)\right] \\ &= \begin{cases} \hat{x}_j[2\ell(1-\hat{x}_i) + 2\hat{x}_i - 1] + (1-\hat{x}_i) & \text{if } \hat{x}_j \leq \hat{x}_i \\ \hat{x}_j[2\hat{x}_i(1-\ell) - 1] + 2\hat{x}_i\ell + (1-\hat{x}_i) & \text{if } \hat{x}_j > \hat{x}_i \end{cases} \end{aligned}$$

As a result, if $\hat{x}_j \notin [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$, then legislators would vote at unanimity to stop at $t = 0$: $p_{i(t=0)} = 1$, $\forall i$, since $U_i(t', \hat{x}_j) - k(t' - t) = \hat{x}_i - k(\ell - t) < \hat{x}_i$ for all $t' > t$.

If $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \bar{x}_j(\ell, \hat{x}_M)]$, at any $t < \ell$, legislator i does not deviate to playing $p_{it} = 1$ if $U_i(\ell, \hat{x}_j) - k(\ell - t) \geq \hat{x}_i$. A necessary and sufficient condition for this to hold at every

$t \in [0, \ell)$ is that it holds at $t = 0$:

$$U_i(\ell, \hat{x}_j) - k\ell \geq \hat{x}_i \Leftrightarrow \begin{cases} \hat{x}_j[2\ell(1-\hat{x}_i) + 2\hat{x}_i - 1] \geq 2\hat{x}_i - 1 + k\ell & \text{if } \hat{x}_j \leq \hat{x}_i \\ \hat{x}_j[2\hat{x}_i(1-\ell) - 1] \geq 2\hat{x}_i(1-\ell) - 1 + k\ell & \text{if } \hat{x}_j > \hat{x}_i \end{cases}$$

Next we prove the following lemma:

Lemma 2. *For any $(\ell, \hat{x}_j)$ such that $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \hat{x}_M]$, a majority of legislators waits until ℓ if and only if:*

$$\hat{x}_j \geq \frac{k\ell + 2\hat{x}_M - 1}{2\ell(1-\hat{x}_M) + 2\hat{x}_M - 1}$$

Proof of Lemma 2.

1. Suppose that $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$. Then $\hat{x}_j(2\ell-1) + 1 - k\ell \geq 2\hat{x}_M(1-\hat{x}_j(1-\ell)) \geq 2\hat{x}_i(1-\hat{x}_j(1-\ell))$ for any $\hat{x}_i \in [\hat{x}_j, \hat{x}_M]$ since $(1-\hat{x}_j(1-\ell)) \geq 0$. Therefore every legislator such that $\hat{x}_i \in [\hat{x}_j, \hat{x}_M]$ waits until ℓ. In addition, $\hat{x}_j(2\ell-1)+1-k\ell \geq 2\hat{x}_i(1-\hat{x}_j(1-\ell))$ for any $\hat{x}_i \in [\hat{x}_j, \hat{x}_M]$ implies that $\hat{x}_j(2\ell-1)+1-k\ell-2\hat{x}_j(1-\hat{x}_j(1-\ell)) \geq 0$, which implies $-2\hat{x}_j(1-\hat{x}_j)(1-\ell)-\hat{x}_j+1-k\ell \geq 0$. Finally, note that $-2\hat{x}_i(1-\hat{x}_j)(1-\ell)-\hat{x}_j+1-k\ell > -2\hat{x}_j(1-\hat{x}_j)(1-\ell)-\hat{x}_j+1-k\ell$ for any $\hat{x}_i < \hat{x}_j$, so $-2\hat{x}_i(1-\hat{x}_j)(1-\ell)-\hat{x}_j+1-k\ell > 0$, which implies $U_i(\ell, \hat{x}_j) - k\ell \geq \hat{x}_i$ for all $\hat{x}_i < \hat{x}_j$.

   Therefore, if $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ then all legislators $\hat{x}_i \in [0, \hat{x}_M]$ wait until ℓ.

2. Suppose that $\hat{x}_j < \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$. Then $\hat{x}_j(2\ell-1) + 1 - k\ell < 2\hat{x}_M(1-\hat{x}_j(1-\ell)) \leq 2\hat{x}_i(1-\hat{x}_j(1-\ell))$, $\forall\ \hat{x}_i \in [\hat{x}_M, 1]$, so a majority votes to stop immediately: $p_{it} = 1$, $\forall t \in [0, 1]$ is an equilibrium strategy for all $\hat{x}_i \in [\hat{x}_M, 1]$. This proves the 'only if' part.

□

We prove a similar result for $\hat{x}_j \geq \hat{x}_M$:

Lemma 3. *For any $(\ell, \hat{x}_j)$ such that $\hat{x}_j \in [\hat{x}_M, \bar{x}_j(\ell, \hat{x}_M)]$, a majority of legislators waits until ℓ if and only if:*

$$\hat{x}_j \leq \frac{1 - k\ell - 2\hat{x}_M(1-\ell)}{1 - 2\hat{x}_M(1-\ell)}$$

Proof of Lemma 3. First note that $\hat{x}_j \leq \bar{x}_j(\ell, \hat{x}_M)$ implies $\hat{x}_j \leq \frac{1}{2(1-\ell)}$, and therefore $2\hat{x}_M(1-\ell) - 1 \leq 0$ for $\hat{x}_M \leq \hat{x}_j$.

1. Suppose that $\hat{x}_j \leq \frac{1-k\ell-2\hat{x}_M(1-\ell)}{1-2\hat{x}_M(1-\ell)}$. This implies that $(1 - 2\hat{x}_M(1-\ell))(1-\hat{x}_j) \geq k\ell$. The left-hand side is decreasing in $\hat{x}_M$, so for any $\hat{x}_i \leq \hat{x}_M$, we also have $(1 - 2\hat{x}_i(1-\ell))(1-\hat{x}_j) \geq k\ell$. The last inequality implies $U_i(\ell, \hat{x}_j) - k\ell \geq \hat{x}_i$ for any $\hat{x}_i \leq \hat{x}_M \leq \hat{x}_j$ so this constitutes a majority who votes to wait.

2. Suppose that $\hat{x}_j > \frac{1-k\ell-2\hat{x}_M(1-\ell)}{1-2\hat{x}_M(1-\ell)}$. Then $(1-2\hat{x}_M(1-\ell))(1-\hat{x}_j) < k\ell$ and therefore $(1-2\hat{x}_i(1-\ell))(1-\hat{x}_j) < k\ell$ for any $\hat{x}_i \in [\hat{x}_M, \hat{x}_j]$. In addition, this implies that at $\hat{x}_i = \hat{x}_j$, $(1-2\hat{x}_j(1-\ell))(1-\hat{x}_j) < k\ell$. In addition, $(1-2\hat{x}_j(1-\ell))(1-\hat{x}_j) < k\ell \Rightarrow 1+\hat{x}_j(2\ell-1)-\hat{x}_j[2\ell\hat{x}_j+2(1-\hat{x}_j)] < k\ell$. Therefore, for any $\hat{x}_i \geq \hat{x}_j$, $1+\hat{x}_j(2\ell-1)-\hat{x}_i[2\ell\hat{x}_j+2(1-\hat{x}_j)] < k\ell$. Re-arranging gives $U_i(\ell,\hat{x}_j)-k\ell \geq \hat{x}_i$ for any $\hat{x}_i \geq \hat{x}_j$. Therefore, all legislators with $\hat{x}_i \geq \hat{x}_M$ vote to stop immediately.

□

A.4.3 Optimal choice of intermediary and timing for lobbyist

The lobbyist's problem is to solve:

$$\begin{aligned}
&\max_{\ell,\hat{x}_j} && \mathbb{P}(x_M = 1) - k\ell = 1 - \hat{x}_j - k\ell \\
&\text{s.t} \quad (1) && \hat{x}_j \leq \bar{x}_j(\ell,\hat{x}_M) \\
&\qquad (2) && \hat{x}_j \geq \underline{x}_j(\ell,\hat{x}_M) \\
&\qquad (3a) && \hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1} \text{ if } \hat{x}_j \leq \hat{x}_M \\
&\qquad (3b) && \hat{x}_j \leq \frac{1-k\ell-2\hat{x}_M(1-\ell)}{1-2\hat{x}_M(1-\ell)} \text{ if } \hat{x}_j > \hat{x}_M
\end{aligned}$$

The first two constraints ensure that the intermediary j makes an informative endorsement that is followed by the median (Lemma 1). The last two ensure that the median is happy to wait to get the information at time l given that the intermediary is j (Lemmas 2 and 3).

We divide the proof into two cases that make it easier to pin down the value of $\underline{x}_l(\ell,\hat{x}_M)$. Recall that $\underline{x}_j(\ell,\hat{x}_M) = \max\left\{\frac{1-2\ell}{2(1-\ell)}, \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}\right\}$ and note that:

$$\frac{1-2\ell}{2(1-\ell)} > \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1} \Leftrightarrow \hat{x}_M < \frac{5}{9} \text{ and } \ell \in \left(\frac{1}{4}-\frac{1}{4}\sqrt{\frac{5-9\hat{x}_M}{1-\hat{x}_M}}, \frac{1}{4}+\frac{1}{4}\sqrt{\frac{5-9\hat{x}_M}{1-\hat{x}_M}}\right)$$

<u>1. Case 1:</u> $\hat{x}_M \geq \frac{5}{9}$.

In this case, $\frac{1-2\ell}{2(1-\ell)} \leq \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ for any ℓ, so $\underline{x}_j(\ell,\hat{x}_M) = \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$. We reduce the problem above with three steps:

1. We first note that it can never be optimal to choose $\hat{x}_j > \hat{x}_M$. Suppose the lobbyist did, then since both $\bar{x}_j(\ell,\hat{x}_M)$ and $\frac{1-k\ell-2\hat{x}_M(1-\ell)}{1-2\hat{x}_M(1-\ell)}$ are increasing in ℓ, these constraints cannot be binding. If they were, the lobbyist could deviate to choosing a lower $\hat{x}_j$ while keeping the duration unchanged. In addition, note that $\hat{x}_j \leq \bar{x}_j(\ell,\hat{x}_M) = \frac{1}{2(1-\ell)}$ implies $\ell \geq \frac{2\hat{x}_j-1}{2\hat{x}_j} \geq \frac{2\hat{x}_M-1}{2\hat{x}_M}$, which implies $\hat{x}_M \geq \underline{x}_j(\ell,\hat{x}_M) = \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$. Therefore, any deviation to some $\hat{x}'_j \geq \hat{x}_M$ would satisfy constraint (2).

2. Second, note that constraint (3a) implies constraint (2) for any $k \geq 0$: if $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$, then $\hat{x}_j \geq \frac{2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$.

3. Finally, note that the set of $\hat{x}_j \in [\underline{x}_j(\ell, \hat{x}_M), \hat{x}_M]$ is non-empty if $\ell \geq \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k}$.

As a result, the lobbyist's problem reduces to:

$$\begin{aligned} &\max_{\ell,\hat{x}_j} && 1-\hat{x}_j-k\ell \\ &\text{s.t} \quad (1') && \hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1} \\ & \qquad\;\; (2') && \ell \geq \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k} \\ & \qquad\;\; (3') && \ell \leq 1 \end{aligned}$$

The objective function is decreasing in $\hat{x}_j$ and in l. Note that the RHS of (1') is decreasing in ℓ while the other constraints are independent of $\hat{x}_j$. Therefore, constraint (1') is always binding. If not, for any ℓ satisfying (2') and (3') the lobbyist could decrease $\hat{x}_j$. In addition, constraints (2') and (3') are never binding at the same time provided that $k < 1-\hat{x}_M$. Therefore, we have three cases: (1') and (2') binding, (1') and (3') binding, or (1') only binding.

If both constraints (1') and (2') are binding, then the optimal choice is:

$$\ell_1^* = \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k} \quad \text{and} \quad \hat{x}_j^* = \hat{x}_M$$

If both constraints (1') and (3') are binding, then the optimal choice is:

$$\ell_3^* = 1 \quad \text{and} \quad \hat{x}_j^* = 2\hat{x}_M - 1 + k$$

If only (1') is binding, we can substitute $\hat{x}_j = \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ into the lobbyist's objective constraint and solve:

$$\max_{\ell} \quad \tilde{U}_L = 1 - \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1} - k\ell$$

The first-order condition gives: $\ell_2^* = \frac{1}{2(1-\hat{x}_M)}\sqrt{\frac{6\hat{x}_M-4\hat{x}_M^2-k(2\hat{x}_M-1)-2}{k}} - \frac{2\hat{x}_M-1}{2(1-\hat{x}_M)}$. The second-order condition is satisfied since: $\frac{\partial^2 U_L(\ell)}{\partial \ell^2} = -\frac{4(1-\hat{x}_M)(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}{(2\hat{x}_M-1+2\ell(1-\hat{x}))^3} \leq 0$, which holds when $2(1-\hat{x}_M) > k$.

Since the unconstrained maximizer always gives a higher payoff than the constrained ones, we can characterize the optimal duration ℓ^*, if it is positive, as follows:

1. If $\ell_2^* < \ell_1^*$, then $\ell^* = \ell_1^* = \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k}$.

2. If $\ell_2^* \in (\ell_1^*, 1)$, then $\ell^* = \ell_2^* = \frac{1}{2(1-\hat{x}_M)}\sqrt{\frac{6\hat{x}_M - 4\hat{x}_M^2 - k(2\hat{x}_M-1)-2}{k}} - \frac{2\hat{x}_M - 1}{2(1-\hat{x}_M)}$.
3. If $\ell_2^* > \ell_3^*$, then $\ell^* = \ell_3^* = 1$.

Finally, we can separate these three cases as a function of k and $\hat{x}_M$:

1. $\ell_1^* < \ell_2^*$ if $\left.\frac{\partial U_L(\ell)}{\partial \ell}\right|_{\ell=\ell_1^*} > 0$. This inequality holds if:

$$\frac{2(k^2\hat{x}_M + 2(1-\hat{x}_M)^2\hat{x}_M^2 - k(1-\hat{x}_M)(4\hat{x}_M - 1))}{(2\hat{x}_M - 1)(2(1-\hat{x}_M) - k)} > 0$$

   If $k < 1 - \hat{x}_M < 2(1-\hat{x}_M)$, the denominator is positive. The numerator is positive if and only if $k < \underline{k}$ or $k > \bar{k}$, where $\underline{k}$ and $\bar{k}$ are the two roots of $k^2\hat{x}_M + 2(1-\hat{x}_M)^2\hat{x}_M^2 - k(1-\hat{x}_M)(4\hat{x}_M - 1)$: $\bar{k} = \frac{1}{2\hat{x}_M}\left((4\hat{x}_M - 1)(1-\hat{x}_M) + (1-\hat{x}_M)\sqrt{(2\hat{x}_M - 1)(6\hat{x}_M - 1 - 4\hat{x}_M^2)}\right)$ and $\underline{k} = \frac{1}{2\hat{x}_M}\left((4\hat{x}_M - 1)(1-\hat{x}_M) - (1-\hat{x}_M)\sqrt{(2\hat{x}_M - 1)(6\hat{x}_M - 1 - 4\hat{x}_M^2)}\right)$. We note that $\bar{k} > 1 - \hat{x}_M$ and that $\underline{k} \in (0, 1-\hat{x}_M)$. Therefore for any $k < 1 - \hat{x}_M$, we have $k < \bar{k}$.

   So the inequality holds whenever $k < \underline{k}$ and in this case $\ell_1^* < \ell_2^*$ implying that $\ell^* = \ell_2^*$ (only constraint (1') binds). If $k > \underline{k}$, the inequality does not hold and $\ell_1^* > \ell_2^*$ implying that $\ell^* = \ell_1^*$ (both constraints (1') and(2') bind).

2. $\ell_2^* < 1$ if $\frac{1}{2(1-\hat{x}_M)}\sqrt{\frac{6\hat{x}_M - 4\hat{x}_M^2 - k(2\hat{x}_M-1)-2}{k}} - \frac{2\hat{x}_M - 1}{2(1-\hat{x}_M)} < 1 \Leftrightarrow \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{\hat{x}_M} < k$.

3. Finally, $\underline{k} > \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{\hat{x}_M}$ since $\frac{(1-\hat{x}_M)}{2\hat{x}_M}\left((4\hat{x}_M - 1) - \sqrt{(2\hat{x}_M - 1)(6\hat{x}_M - 1 - 4\hat{x}_M^2)}\right) > \frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{\hat{x}_M}$.

   For $k > 1 - \hat{x}_M$, the median would not be willing to wait even if the information was given to her directly and even if $\ell = 1$, so the optimal choice for the lobbyist is $\ell^* = 0$.

Thus, let:

$$\begin{aligned}
k_1(\hat{x}_M) &= \frac{(1-\hat{x}_M)(2\hat{x}_M - 1)}{\hat{x}_M} \\
k_2(\hat{x}_M) &= \frac{(1-\hat{x}_M)}{2\hat{x}_M}\left((4\hat{x}_M - 1) - \sqrt{(2\hat{x}_M - 1)(6\hat{x}_M - 1 - 4\hat{x}_M^2)}\right) \\
k_3(\hat{x}_M) &= 1 - \hat{x}_M
\end{aligned}$$

We get the following result:

1. If $k < k_1(\hat{x}_M)$, then $\ell^* = \ell_3^* = 1$ and $\hat{x}_j^* = 2\hat{x}_M - 1 + k$.
2. If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$ then $\ell^* = \ell_2^*$ and $\hat{x}_j^* = \frac{k}{2(1-\hat{x}M)}\left(1 + \sqrt{\frac{(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}{k}}\right)$.
3. If $k \in (k_2(\hat{x}_M), k_3(\hat{x}_M))$ then $\ell^* = \ell_1^*$ and $\hat{x}_j^* = \hat{x}_M$.
4. If $k > k_3(\hat{x}_M)$, then $\ell^* = 0$ and any $\hat{x}_j^* \in [0, 1]$ is an equilibrium.

Finally, note that for any $k \leq 1 - \hat{x}_M$, the lobbyist's utility is positive. When $\ell^* = \ell_2^*$, we know that $\tilde{U}_L(\ell)$ is maximised over $\ell \in [0,1]$. Since $\ell = 0$ is available, and $U_L(\ell = 0) = 0$, so $\tilde{U}_L(\ell_2^*) \geq 0$. When $\ell^* = 1$, $U_L(1) = 1 - \hat{x}_j(1) - k = 2((1-\hat{x}_M) - k) > 0$ since $k < 1 - \hat{x}_M$. When $\ell^* = \ell_1^*$, $U_L(\ell_1^*) = 1 - \hat{x}_M - k\ell_1^* > 0$ since $k < 1 - \hat{x}_M$ and $\ell_1^* \leq 1$.

1. Case 2: $\hat{x}_M < \frac{5}{9}$.

In this case, $\frac{1-2\ell}{2(1-\ell)} > \frac{2\hat{x}_M - 1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ if $\ell \in \left(\frac{1}{4} - \frac{1}{4}\sqrt{\frac{5-9\hat{x}_M}{1-\hat{x}_M}}, \frac{1}{4} + \frac{1}{4}\sqrt{\frac{5-9\hat{x}_M}{1-\hat{x}_M}}\right)$. The additional constraint imposes new restrictions on the waiting constraint as well. Let

$$\underline{\ell} = \frac{1 - k - \hat{x}_M - \sqrt{5 - 4k + k^2 - 14\hat{x}_M + 6k\hat{x}_M + 9\hat{x}_M^2}}{2(2(1-\hat{x}_M) - k)}$$
$$\bar{\ell} = \frac{1 - k - \hat{x}_M + \sqrt{5 - 4k + k^2 - 14\hat{x}_M + 6k\hat{x}_M + 9\hat{x}_M^2}}{2(2(1-\hat{x}_M) - k)}$$

We have $\frac{1-2\ell}{2(1-\ell)} > \frac{k\ell + 2\hat{x}_M - 1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ if: $\ell \in (\underline{\ell}, \bar{\ell})$. We recover a similar characterization as in the previous case.

1. If $k < k_1(\hat{x}_M)$, then $\ell^* = 1$. To see this, note that for any $k < k_1(\hat{x}_M)$, $\ell_2^* > 1$, and that $1 > \bar{\ell}$ for any $k < 1 - \hat{x}_M$ and $\hat{x}_M < \frac{5}{9}$. Therefore, $\ell^* = 1$ is never in the range where the constraint $\hat{x}_j \geq \frac{1-2\ell}{2(1-\ell)}$ binds.
2. If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$, then we need to consider two cases:
   (a) Either the constraint $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ binds first. This gives the same solution as when $\hat{x}_M \geq \frac{5}{9}$: ℓ_2^*. Given this solution, the constraint above is indeed the binding one provided that: $\ell_2^* \in (\underline{\ell}, \bar{\ell})$. Since $\ell_2^* > \underline{\ell}$ for any $k < 1 - \hat{x}_M$ and $\hat{x}_M < \frac{5}{9}$, we only need to find when $\ell_2^* < \bar{\ell}$. This is the case for a range of $k \in (k_{2A}(\hat{x}_M), k_{2B}(\hat{x}_M) \subset [k_1(\hat{x}_M), k_2(\hat{x}_M)]$. Therefore, for $k \leq k_{2A}(\hat{x}_M)$ or $k \in [k_{2B}(\hat{x}_M), k_2(\hat{x}_M)]$, then:

   $$\ell^* = \ell_2^* \quad \text{and} \quad \hat{x}_j^* = \frac{k}{2(1-\hat{x}_M)}\left(1 + \sqrt{\frac{(2\hat{x}_M - 1)(2(1-\hat{x}_M) - k)}{k}}\right)$$

   (b) Or the constraint $\hat{x}_j \geq \frac{1-2\ell}{2(1-\ell)}$ binds first. In that case, substituting the constraint in the lobbyist's objective function, we find that the objective function is everywhere increasing in ℓ, so the optimal solution is:

   $$\ell^* = \bar{\ell} \quad \text{and} \quad \hat{x}_j^* = \frac{k + 3\hat{x}_M - 1 - \sqrt{5 - 4k + k^2 - 14\hat{x}_M + 6k\hat{x}_M + 9\hat{x}_M^2}}{2}$$

   We note that this solution satisfies $\ell^* \in (0,1)$ and $\hat{x}_j^* \in (0, \hat{x}_M)$.

   Finally, note that $\bar{\ell} > \underline{\ell}$ if and only if $k < 2 - 3\hat{x}_M - \sqrt{2\hat{x}_M - 1}$. Therefore, if this condition is not satisfied, then there is no ℓ such that the constraint $\hat{x}_j \geq \frac{1-2\ell}{2(1-\ell)}$

can bind.

3. If $k > k_2(\hat{x}_M)$, then $k > 2-3\hat{x}_M - \sqrt{2\hat{x}_M - 1}$ for any $\hat{x}_M \leq \frac{5}{9}$, so the set $[\underline{\ell}, \bar{\ell}]$ is empty and the solutions are the same as when $\hat{x}_M \geq \frac{5}{9}$.

A.5 Proof of Propositions 2 and 3

If $k < k_1(\hat{x}_M)$ or $k > k_3(\hat{x}_M)$, then clearly $\frac{\partial \ell^*}{\partial k} = 0$. If $k < k_1(\hat{x}_M)$, then $\frac{\partial \hat{x}_j^*}{\partial k} = 1 > 0$ whereas if $k > k_3(\hat{x}_M)$, $\frac{\partial \hat{x}_j^*}{\partial k} = 0$. If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$ and $\ell^* = \ell_2^*$, then: $\frac{\partial \ell^*}{\partial k} = \frac{\partial \ell_2^*}{\partial k} = -\left(\frac{2\hat{x}_M - 1}{2k^2\sqrt{\frac{(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}{k}}}\right) < 0$ and for the intermediary, given $k < 1-\hat{x}_M$:

$$\frac{\partial \hat{x}_j^*}{\partial k} = \frac{(1-\hat{x}_M-k)(2\hat{x}_M-1) + \sqrt{k(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}}{2(1-\hat{x}_M)\sqrt{k(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}} > 0$$

If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$ and $\ell^* = \bar{\ell}$, then, $\forall \hat{x}_M < \frac{5}{9}$ and $\forall k < 1-\hat{x}_M$: $\frac{\partial \ell^*}{\partial k} = \frac{\partial \bar{\ell}}{\partial k} = -\left(\frac{1-\hat{x}_M - \frac{1-\hat{x}_M(4-k-3\hat{x}_M)}{\sqrt{5-4k+k^2-14\hat{x}_M+6k\hat{x}_M+9\hat{x}_M^2}}}{2(2(1-\hat{x}_M)-k)^2}\right) < 0$ and given $k < 2-3\hat{x}_M - \sqrt{2\hat{x}_M-1}$:

$$\frac{\partial \hat{x}_j^*}{\partial k} = \frac{2-3\hat{x}_M - k + \sqrt{5-4k+k^2-14\hat{x}_M+6k\hat{x}_M+9\hat{x}_M^2}}{2\sqrt{5-4k+k^2-14\hat{x}_M+6k\hat{x}_M+9\hat{x}_M^2}} > 0$$

If $k \in (k_2(\hat{x}_M), k_3(\hat{x}_M))$, then $\frac{\partial \ell^*}{\partial k} = \frac{\partial \ell_1^*}{\partial k} = \frac{(2\hat{x}_M-1)(1-\hat{x}_M)}{(2\hat{x}_M(1-\hat{x}_M)-k)^2} > 0$, and $\frac{\partial \hat{x}_j^*}{\partial k} = 0$.

A.6 Proof of Proposition 4

The equilibrium probability of choosing the correct policy for the median, given that she follows the intermediary's advice is:

$$\begin{aligned}
F(k, \hat{x}_M) &= \mathbb{P}(m_j = 1)\,\mathbb{P}(\omega \geq \hat{x}_M | m_j = 1) + \mathbb{P}(m_j = 0)\,\mathbb{P}(\omega < \hat{x}_M | m_j = 0) \\
&= \ell\left[\mathbb{P}(\omega \geq \hat{x}_j)\frac{\mathbb{P}(\omega \geq \hat{x}_M \cap \omega \geq \hat{x}_j)}{\mathbb{P}(\omega \geq \hat{x}_j)} + \mathbb{P}(\omega < \hat{x}_j)\frac{\mathbb{P}(\omega < \hat{x}_M \cap \omega < \hat{x}_j)}{\mathbb{P}(\omega < \hat{x}_j)}\right] \\
&+ (1-\ell)\left[\mathbb{P}(\tilde{s} \geq \hat{x}_j)\,\mathbb{P}(\omega \geq \hat{x}_M) + \mathbb{P}(\tilde{s} < \hat{x}_j)\,\mathbb{P}(\omega < \hat{x}_M)\right] \\
&= \ell \times 2(1-\hat{x}_M)\hat{x}_j + (1-\hat{x}_M)(1-\hat{x}_j) + \hat{x}_j\hat{x}_M \qquad (4)
\end{aligned}$$

Substituting the optimal duration and intermediary in this expression gives:

1. If $k < k_1(\hat{x}_M)$, $F(k, \hat{x}_M) = k + \hat{x}_M$, so $\frac{\partial F(k, \hat{x}_M)}{\partial k} = 1 > 0$.
2. If $k \in (k_1(\hat{x}_M), k_2(\hat{x}_M))$ and $\ell = \ell_2^*$,

$$F(k, \hat{x}_M) = \hat{x}_M + k + \frac{k}{2(1-\hat{x}_M)}\left[\sqrt{\frac{(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}{k}} - 1\right]$$

$$\text{So} \quad \frac{\partial F(k,\hat{x}_M)}{\partial k} = \frac{(2\hat{x}_M-1)(1-\hat{x}_M-k-\sqrt{k(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}}{2(1-\hat{x}_M)\sqrt{k(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)}}$$

This is positive if and only if:

$$1-\hat{x}_M-k \geq \sqrt{k(2\hat{x}_M-1)(2(1-\hat{x}_M)-k)} \Leftrightarrow (1+4k)\hat{x}_M^2 - 2(1+2k-k^2)\hat{x}_M + 1 \geq 0$$

This quadratic in $\hat{x}_M$ has two roots, but the larger root is greater than $1-k$, so $\hat{x}_M$ is always lower than the higher root. Therefore, the inequality holds if any only if $\hat{x}_M$ is lower than the lowest root, that is: $\hat{x}_M \leq \frac{1+2k-k^2-\sqrt{2k^2-4k^3+k^4}}{1+4k}$. Similarly, we can show that $F(k,\hat{x}_M)$ is non-monotonic in k when $\ell^* = \bar{\ell}$.

3. If $k \in (k_2(\hat{x}_M), k_3(\hat{x}_M))$, $F(k,\hat{x}_M) = k+\hat{x}_M - \frac{k(1-\hat{x}_M-k)}{2(1-\hat{x}_M)\hat{x}_M-k}$ so $\frac{\partial F(k,\hat{x}_M)}{\partial k} = \frac{(2\hat{x}_M-1)\hat{x}_M(1-\hat{x}_M)^2}{(2(1-\hat{x}_M)\hat{x}_M-k)^2} \geq 0$.
4. If $k > k_3(\hat{x}_M)$, $F(k,\hat{x}_M) = \hat{x}_M$, so $\frac{\partial F(k,\hat{x}_M)}{\partial k} = 0$.

B Proofs of extensions and robustness

B.1 Extension: No commitment to duration

Consider a variant of the model in which at every point in time $t \in [0,1]$, the lobbyist can decide to stop the investigation. We prove the following proposition:

Proposition 5. *If the lobbyist cannot commit to ℓ and the cost of waiting is sufficiently low such that $k \leq \hat{x}_M - \sqrt{2\hat{x}_M-1}$, then the lobbyist buys access to intermediary j with*

$$\hat{x}_j^* = \frac{1}{2}\left(1+\hat{x}_M-k-\sqrt{1+k^2-\hat{x}_M(2+2k-\hat{x}_M)}\right).$$

and conducts an investigation that lasts $\ell^* = \frac{1+k-3\hat{x}_M+\sqrt{1+(k-\hat{x}_M)^2-2\hat{x}_M}}{2(k-2\hat{x}_M)}$. If $k > \hat{x}_M - \sqrt{2\hat{x}_M-1}$, then there is no lobbying. When there is lobbying, the intermediary's threshold $\hat{x}_j^*$, the duration ℓ^* and the quality of policy are all increasing in k.

Proof of Proposition 5. Given an equilibrium conjecture (j,ℓ), the median must find it optimal to wait. There are two cases, depending on whether $\hat{x}_j < \hat{x}_M$ or $\hat{x}_j \geq \hat{x}_M$.

Case 1. $\hat{x}_j \leq \hat{x}_M$**.** By Lemma 2, the median waits if and only if $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$. Hence, if the lobbyist selects j with $\hat{x}_j < \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$, a majority will not wait for the lobbyist's investigation, and the lobbyist earns a payoff of 0. Alternatively, if $\hat{x}_j \in \left[\frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}, \hat{x}_M\right]$, then a majority is willing to wait. At time $\ell \in (0,1]$ a majority would be convinced if both $\hat{x}_j \geq 2\hat{x}_M-1$ and $\ell \geq \frac{1-2\hat{x}_M+\hat{x}_j(2\hat{x}_M-1)}{2\hat{x}_j\hat{x}_M-2\hat{x}_j}$. For each $\hat{x}_j \in \left[\frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}, \hat{x}_M\right]$ for which a majority potentially would be willing to wait, the lobbyist stops the investigation if $\ell^* = \frac{1-2\hat{x}_M+\hat{x}_j(2\hat{x}_M-1)}{2\hat{x}_j\hat{x}_M-2\hat{x}_j}$. The lobbyist's payoff is 0 if $\ell < \ell^*$, and decreases for any $l > \ell^*$ as the lobbyist pays a cost of waiting without increasing the probability of getting its most preferred policy. However, for any $\hat{x}_j$ with $\ell^* = \frac{1-2\hat{x}_M+\hat{x}_j(2\hat{x}_M-1)}{2\hat{x}_j\hat{x}_M-2\hat{x}_j}$, the median is indifferent between accepting or rejecting at time ℓ. Hence, at time $t=0$, we need the median to be willing to wait: $\hat{x}_j \geq \frac{k\ell^*+2\hat{x}_M-1}{2\ell^*(1-\hat{x}_M)+2\hat{x}_M-1}$, which is a contradiction. Thus, no equilibrium exists with intermediary $\hat{x}_j \leq \hat{x}_M$.

Case 2. $\hat{x}_j > \hat{x}_M$. To ensure legislator j gives a positive endorsement, it is necessary that both $s \geq \hat{x}_j$, and $\ell \geq 1 - \frac{1}{2\hat{x}_j}$. Once legislator j gives such a positive endorsement, this is persuasive for the median (and hence, a majority) as $s \geq \hat{x}_M$ and $\ell \geq 1 - \frac{1}{2\hat{x}_M}$. Given that waiting is costly, the lobbyist will stop investigating as soon as the signal is persuasive for intermediary j, i.e., $\ell(\hat{x}_j) = 1 - \frac{1}{2\hat{x}_j}$ To ensure that the median is willing to wait for further information, we require that the following condition is met, which follows from Lemma 3.

$$\hat{x}_j \leq \frac{1 - k\ell(\hat{x}_j) - 2\hat{x}_M(1 - \ell(\hat{x}_j))}{1 - 2\hat{x}_M(1 - \ell(\hat{x}_j))}.$$

If $k \leq \hat{x}_M - \sqrt{2\hat{x}_M - 1}$, then the set of $\hat{x}_j$ such that the above inequality holds is

$$\hat{x}_j \in \left[\frac{1 + \hat{x}_M - k - \sqrt{1 + k^2 - \hat{x}_M(2 + 2k - \hat{x}_M)}}{2}, \frac{1 + \hat{x}_M - k + \sqrt{1 + k^2 - \hat{x}_M(2 + 2k - \hat{x}_M)}}{2}\right]$$

Otherwise, if $k > \hat{x}_M - \sqrt{2\hat{x}_M - 1}$, this set is empty. Thus, if $k \leq \hat{x}_M - \sqrt{2\hat{x}_M - 1}$, the optimal $\hat{x}_j$ is the minimum of the set of $\hat{x}_j$'s that satisfies the inequality, as that increases the probability of the lobbyist getting his most preferred policy and minimizes the cost of waiting subject to the constraint. Hence, if $k \leq \hat{x}_M - \sqrt{2\hat{x}_M - 1}$, then $\hat{x}_j = \frac{1}{2}\left(1 + \hat{x}_M - k - \sqrt{1 + k^2 - \hat{x}_M(2 + 2k - \hat{x}_M)}\right)$, and otherwise there is no lobbying. $\square$

B.2 Extension: No commitment to intermediary

Consider the following modification to our original model. Instead of committing to some $\hat{x}_j$ at the start of the game, the lobbyist can choose with whom to share information at any point once the investigation ends. For simplicity, we assume that the lobbyist can only make this choice at $t = \ell$.[11] We also assume that the legislators can see who has been targeted at time $t = \ell^*$, but that every legislator can make an endorsement.[12]

Formally, the lobbyist's strategy is now a pair $(\ell, \gamma(s))$ where $\gamma : [0, 1] \to [0, 1] \cup \emptyset$ maps a signal realization to a choice of intermediary with whom to share the evidence (where $\emptyset$

[11] Choosing the intermediary before would carry no information as the lobbyist has not observed a signal and could choose a different intermediary later. Choosing the intermediary later would lead to costly delays for all players but could, in principle, be used by the lobbyist to signal some information. However, we conjecture that whatever information is transmitted in such an equilibrium could be replicated by an equilibrium in which the lobbyist chooses the intermediary at $t = \ell$.

[12] An alternative would be to assume that the legislators do not observe which legislator has been targeted but that only the legislator who has been targeted can make an endorsement. Both assumptions allow us to sustain the same equilibria. Our assumption simply makes it easier to ensure that the beliefs formed by legislators upon observing the legislator selected are consistent with the beliefs formed upon hearing the endorsement, while maintaining the assumption of sincere endorsements. Finally, we could also recover the same equilibria by assuming that all legislators can make endorsements and that the legislators not targeted cannot observe who has been targeted. This would require a more complex characterization where legislators potentially draw inferences from the patterns of endorsements they observe.

denotes sharing the information publicly). Let $\tilde{\mu}_i(s|j)$, $\tilde{\mu}_i : [0,1] \to \Delta[0,1]$, denote the belief formed by legislator i about the cumulative distribution function of s upon observing that legislator j has been targeted, and $\mu_i(s|m_j, j)$, $\mu_i : [0,1] \to \Delta[0,1]$, denote this belief updated following endorsement m_j from legislator j.

Proposition 6. *The following strategy profile and belief function is an equilibrium of the modified game:*

1. *The lobbyist chooses $\ell = \ell^*(k)$, where $\ell^*(k)$ is the equilibrium duration in the original game given some k, as defined in Proposition 1.*
2. *The lobbyist's targeting strategy is $\gamma^*(s) = \hat{x}_j^*(k)$, $\forall\ s \in [0,1]$, where $\hat{x}_j^*(k)$ is the equilibrium intermediary in the original game given k, as defined in Proposition 1.*
3. *Legislators with thresholds $\hat{x}_i \in [0, \hat{x}_M]$ vote $p_{it} = 0$ for any $t < \ell$ and $p_{it} = 1$ for any $t \geq \ell$. Legislators with thresholds $\hat{x}_i \in (\hat{x}_M, 1]$ vote $p_{it} = 0$ for any $t < \underline{t}(\hat{x}_j)$ and $p_{it} = 1$ for any $t \geq \underline{t}(\hat{x}_j)$, for some threshold $\underline{t}(\hat{x}_j) \in (0,1)$.*
4. *All legislators $\hat{x}_j$ give sincere endorsements $m_j = 1$ if $s \geq \hat{x}_j$ and $m_j = 0$ otherwise.*
5. *Legislators with $\hat{x}_i \leq \bar{x}$ vote $x = 1$ if and only if $m_{j^*} = 1$ and legislators with $\hat{x}_i > \bar{x}$ vote $x = 0$ for any m_{j^*}, for some threshold $\bar{x} \in (\hat{x}_M, 1]$.*
6. *Legislators form the following beliefs upon observing that j was targeted:*
   (a) *If $j = j^*$, $\tilde{\mu}_i(s|j = j^*)$ is the prior: $s \sim U[0,1]$.*
   (b) *If $j \neq j^*$, $\tilde{\mu}_i(s|j \neq j^*)$ is such that $\mathbb{P}(s \in \{0, \hat{x}_j\}) = 1$.*
7. *Legislators form beliefs $\mu_i(s|m_j)$ given m_j such that:*
   (a) *If $j = j^*$, $\mu_i(s|m_{j^*} = 0)$ is the uniform distribution on $[0, \hat{x}_j^*)$ and $\mu_i(s|m_{j^*} = 1)$ is the uniform distribution on $[\hat{x}_j^*, 1]$.*
   (b) *If $j \neq j^*$, $\mathbb{P}(s = 0|m_j = 0) = 1$ and $\mathbb{P}(s = \hat{x}_j|m_j = 1) = 1$.*

Proof of Proposition 6. We proceed in three steps: we show that the lobbyist has no incentives to deviate given the legislators' strategies, that the legislators' behavior is consistent with our equilibrium concept given the lobbyist's strategy and that the beliefs they form on path are consistent with Bayes rule and satisfy the intuitive criterion off-path.

1. **For lobbyist:** given the legislators' behavior, the lobbyist expects the process to continue until $t = \ell^*$ and to share information with intermediary j^*. The lobbyist's expected payoff is therefore the same as in the equilibrium characterized in Proposition 1, which we showed is the highest possible payoff for him subject to the constraint that the median waits for information. If the lobbyist deviates to a lower ℓ a majority of legislators would vote to end the process immediately. This gives the lobbyist a payoff of 0. The lobbyist also does not deviate to a higher ℓ as this does not change the equilibrium probability of persuading but increases the waiting costs.

   If at time ℓ^*, the lobbyist deviates to share information s with $j \neq j^*$, then either:

(a) The signal was $s < \min\{\hat{x}_j, \hat{x}_{j^*}\}$. Then j recommends $m_j = 0$ and j^* would have recommended $m_{j^*} = 0$, so in both cases a majority of other legislators believe that $\mathbb{P}(\omega < \hat{x}_i | m, j) < \frac{1}{2}$ and vote for $x = 0$.

(b) The signal was $\hat{x}_j \leq s < \hat{x}_j^*$. Then j recommends $m_j = 1$ and j^* would have recommended $m_{j^*} = 0$. Given off-equilibrium beliefs $\mathbb{P}(s = \hat{x}_j | m_j = 1) = 1$, the median, and a majority of legislators believe that $\mathbb{P}(\omega < \hat{x}_i | m_j = 1) < \frac{1}{2}$ and vote for $x = 0$. So the lobbyist does not gain from deviating.

(c) The signal was $\hat{x}_j < \hat{x}_{j^*} \leq s$. Then j recommends $m_j = 1$ and j^* would have recommended $m_{j^*} = 1$. Given off-equilibrium beliefs $\mathbb{P}(s = \hat{x}_j | m_j = 1) = 1$, and given $\hat{x}_j < \hat{x}_{j^*} \leq \hat{x}_M$, the median, and a majority of legislators believe that $\mathbb{P}(\omega < \hat{x}_i | m_j = 1) < \frac{1}{2}$ and vote for $x = 0$. So the lobbyist actually loses from deviating.

(d) The signal was $\hat{x}_{j^*} \leq s < \hat{x}_j$, then j recommends $m_j = 0$ and j^* would have recommended $m_{j^*} = 1$. So the lobbyist loses from deviating.

(e) The signal was $\hat{x}_{j^*} < \hat{x}_j \leq s$, then j recommends $m_j = 1$ and j^* would have recommended $m_{j^*} = 1$. If $\hat{x}_j \geq \hat{x}_M$ then given off-equilibrium beliefs $\mathbb{P}(s = \hat{x}_j | m_j = 1) = 1$ a majority votes for $x = 1$ but would have voted $x = 1$ in equilibrium anyway. And if $\hat{x}_j < \hat{x}_M$ then given off-equilibrium beliefs $\mathbb{P}(s = \hat{x}_j | m_j = 1) = 1$ a majority votes for $x = 0$. So the lobbyist loses from deviating.

2. **No incentives to deviate for legislators:**

- No deviation from procedural vote: given the lobbyist's strategy, the legislators' procedural vote strategy on path is optimal for the same reasons as in the proof of Proposition 1. Off path, a deviation can only be detected at some $t \geq \ell^*$, so the legislators also have no incentives to deviate at any $t < \ell^*$. At $t > \ell^*$, there is no additional information than at $t = \ell^*$, no matter what beliefs the legislators form about ω following the deviation. Therefore, the legislators expect to support the same policy at any $t \geq \ell^*$ and have no incentives to delay the policy vote.

- No deviation from policy vote: **On path:** the voters form the same beliefs and have the same expected utility as in the proof of Proposition 1 so have no incentives to support another policy if the lobbyist sticks to his strategy.

  Off path: given the beliefs that the legislators form, we get several scenarios depending on which legislator other than j^* shares the endorsement. If $m_j = 0$, then every legislator believes that $s = 0$ and votes for $x = 0$. If $m_j = 1$,

  (a) If $\hat{x}_j \in [0, \hat{x}_M)$, then every legislator believes that $s = \hat{x}_j < \hat{x}_M$ so all legislators with $\hat{x}_i \in [0, \hat{x}_j]$ vote for $x = 1$ and all legislators with $\hat{x}_i \in (\hat{x}_j, 1]$ (a majority) vote for $x = 0$. Note that at ℓ^* the signal is precise enough to persuade the median and therefore precise enough to persuade all legislators below the median.

  (b) If $\hat{x}_j \in [\hat{x}_M, 1]$, then every legislator believes that $s = \hat{x}_j > \hat{x}_M$ so all legislators with $\hat{x}_i \in [0, \bar{x}(\ell^*)]$ (a majority) vote for $x = 1$ and all legislators with $\hat{x}_i \in [\bar{x}(\ell^*), 1]$ vote for $x = 0$, where $\bar{x}(\ell^*) > \hat{x}_M$ is the value of $\hat{x}_i$ such that $\mathbb{P}(\omega \geq \hat{x}_i | s = \hat{x}_j, \ell^*) = \frac{1}{2}$.

3. **On-path beliefs:** when the lobbyist sticks to his equilibrium strategy, beliefs are updated according to Bayes rule. The legislators form the same beliefs about s given m_{j^*} and the same beliefs about ω given their beliefs about s that they formed in Proposition 1. In particular, they learn nothing from the fact that the endorsement comes from legislator j^* because the lobbyist's strategy is to share information with j^* for any $s \in [0, 1]$.

4. **Off-path beliefs:** given that our equilibrium concept is Weak Perfect Bayesian Equilibrium, there is no restriction on off-path beliefs, so the beliefs imposed in this equilibrium are admissible. However, we can also show that these beliefs satisfy the intuitive criterion (Cho and Kreps 1987). The intuitive criterion would be violated if the off-path beliefs put weight on a type of the lobbyist (some s), following a deviation to some $j \neq j^*$ such that this deviation always gives a lower payoff to this type of the lobbyist, *for any* beliefs that the legislators may form. When $j \neq j^*$, the legislators only put weight on type $s = 0$ or type $s = \hat{x}_j$. If following this deviation, the legislators believed that $s = 1$ with probability 1, then this deviation would indeed be profitable for type $s = 0$ and type $s = \hat{x}_j$. Therefore believing $\mathbb{P}(s \in \{0, \hat{x}_j\})$ with probability 1 satisfies the intuitive criterion.

□

Next, we show that this equilibrium gives the lobbyist the highest equilibrium payoff among all equilibria, and that the median receives the same payoff in all these equilibria.

Proposition 7. *In every equilibrium of the modified game, the median legislator receives the same payoff and the lobbyist's payoff is weakly lower than in the equilibrium characterized in Proposition 6.*

Proof of Proposition 7. **For the lobbyist:** any payoff in an equilibrium of the modified game is a payoff that could have been achieved by the lobbyist in the game with commitment. Since the lobbyist chooses the duration and intermediary that maximizes his payoff in the game with commitment, it must be that his payoff in every equilibrium of the modified game is weakly lower than this payoff. This implies that the equilibrium characterized in Proposition 6 is the sender-preferred equilibrium of the modified game.

For the median: in the modified game, the lobbyist commits to a duration at the start of the game, anticipating the equilibrium strategies that will be played in the rest of the game. Consider the duration and intermediary chosen in such an equilibrium $(\ell^*, \hat{x}_j^*)$. In any equilibrium where $\ell^* > 0$, we must have $\hat{x}_j^* \in [2\hat{x}_M - 1 - k, \hat{x}_M]$. If $\hat{x}_j^* < 2\hat{x}_M - 1 - k$, then for any $\ell^* \leq 1$, a majority votes to stop the process at $t = 0$. If $\hat{x}_j^* > \hat{x}_M$, then the lobbyist would deviate to disclosing the information publicly whenever he observes $s \in [\hat{x}_M, \hat{x}_j^*]$. Given this range of intermediaries, suppose by contradiction that $\mathbb{E}[U_M(\ell^*, \hat{x}_j^*)] - k\ell^* > \hat{x}_M$. then the lobbyist could deviate to some $\ell(k) \leq \ell' < \ell^*$, where $\ell(k)$ is the value such that $\mathbb{E}[U_M(\ell(k), \hat{x}_j^*)] - k\ell(k) = \hat{x}_M$, without affecting (1) the procedural vote strategies, (2) the endorsement strategy, (3) the belief updating. The only difference would be that, at time $t = \ell'$, a smaller group of legislators would support policy $x = 1$ following $m_j = 1$, but this group would still constitute a majority. This would therefore be a profitable deviation for the lobbyist. If $\mathbb{E}[U_M(\ell^*, \hat{x}_j^*)] - k\ell^* < \hat{x}_M$, a majority of legislators would vote to stop the process immediately at $t = 0$, so the lobbyist would deviate to a larger ℓ. Finally, in any equilibrium where $\ell^* = 0$, the median receives a payoff of $\hat{x}_M$. Therefore, in any equilibrium of the modified game, the median receives a payoff of $\hat{x}_M$. □

B.3 Extension: Internal information acquisition

Consider a variant of our baseline model in which neither the legislature nor the lobbyist face a cost of waiting: $k = 0$. Instead, the legislators can obtain information from an internal investigation as follows: if the legislators stop the investigation at time $t \leq \min\left\{\frac{1}{\alpha}, 1\right\}$, they observe the state with probability αt and observe nothing with probability $1 - \alpha t$, where $\alpha > 0$. For tractability, and to avoid trivial cases, we impose that the legislators must stop their investigation before the lobbyist's one ends: $t \in [0, \ell]$. We also restrict attention to the case $\hat{x}_M \in (\frac{5}{9}, 1)$. Finally, we assume that, when indifferent between different stopping times, the legislators stop their investigation at the earliest time.

As in the baseline model, the lobbyist moves first and chooses a length of investigation ℓ and an intermediary $\hat{x}_j$. The legislators move next and choose at which time t to stop their investigation given ℓ and $\hat{x}_j$.

We first solve for the median's best response. Given that waiting is free and that the median's expected utility is strictly increasing in t for any $t < \min\{\frac{1}{\alpha}, \ell\}$, the median never stops the investigation at any $t < \ell$. Moreover, the median stops immediately as soon as she can observe the state with probability 1, which implies that $t = \min\{\frac{1}{\alpha}, \ell\}$.

Next, we can solve for the lobbyist's strategy given this best response. First note that if the lobbyist chooses some $\ell > \frac{1}{\alpha}$, then his utility is $1 - \hat{x}_M$ independently of ℓ and $\hat{x}_j$. If $\ell < \frac{1}{\alpha}$, then $t^* = \ell$, so the median observes the state with probability $\alpha\ell$, in which case the lobbyist gets his preferred policy with probability $1 - \hat{x}_M$. With remaining probability, the lobbyist earns $1 - \hat{x}_j$ as long as the intermediary is persuasive.

The lobbyist solves the following maximization problem:

$$\max_{\hat{x}_j, \ell} \ \alpha\ell(1 - \hat{x}_M) + (1 - \alpha\ell)(1 - \hat{x}_j)$$
$$\text{s.t. } \hat{x}_j \geq \max\left\{\frac{2\hat{x}_M - 1}{2\hat{x}_M - 1 + 2\ell(1 - \hat{x}_M)}, \frac{1 - 2\ell}{2(1 - \ell)}\right\} \text{ and } \ell \in \left[\frac{2\hat{x}_M - 1}{2\hat{x}_M}, \frac{1}{\alpha}\right]$$

The first constraint ensures that the lobbyist's intermediary is persuasive and that the the intermediary only gives a positive endorsement if $s \geq \hat{x}_j$ (as in the Proof of Lemma 2). Given the assumption that $\hat{x}_M > \frac{5}{9}$, satisfying the first condition implies that the second is satisfied. The second condition ensures the existence of an intermediary with $\hat{x}_j \leq \hat{x}_M$ who is persuasive. Solving this maximization problem leads to the following proposition:

Proposition 8. *For $\hat{x}_M \in (\frac{5}{9}, 1)$, the equilibrium intermediary and length of investigation are as follows:*

- *If $\alpha \leq \frac{4\hat{x}_M - 2}{4\hat{x}_M - 1}$, then $\hat{x}_j^* = 2\hat{x}_M - 1$ and $\ell^* = 1$.*
- *If $\alpha \in \left(\frac{4\hat{x}_M - 2}{4\hat{x}_M - 1}, \frac{2\hat{x}_M}{2\hat{x}_M - 1}\right)$, then $\hat{x}_j^* = \frac{\alpha(2\hat{x}_M - 1)\hat{x}_M}{\sqrt{\alpha\hat{x}_M(1 - 2\hat{x}_M)(2 - \alpha - 2(1 - \alpha)\hat{x}_M}}$ and*

$\ell^* = 1 + \frac{1}{2}\left(\frac{\sqrt{\alpha\hat{x}_M(2\hat{x}_M-1)(2-\alpha-2(1-\alpha)\hat{x}_M}}{\alpha(1-\hat{x}_M)\hat{x}_M} - \frac{1}{1-\hat{x}_M}\right)$.

- *If* $\alpha \geq \frac{2\hat{x}_M}{2\hat{x}_M-1}$, *then* $\hat{x}_j^*$ *is irrelevant and* $\ell^* \geq \frac{1}{\alpha}$.

If the legislature has weak internal information acquisition capacity, the lobbyist's chooses the longest possible investigation and a very friendly intermediary. This is the same strategy the lobbyist uses if the legislators cannot acquire any information ($\alpha = 0$). Above a threshold $\alpha = \frac{4\hat{x}_M-2}{4\hat{x}_M-1}$, the lobbyist internalizes the fact that if he waits too long, it becomes more likely that the legislature discovers the true state and ignores the lobbyist's information. As a result, when α increases, the lobbyist is forced to shorten his investigation. The shorter investigation means that the lobbyist must choose a more moderate intermediary to ensure that his information remains persuasive. Above a second threshold, $\frac{2\hat{x}_M}{2\hat{x}_M-1} > 1$, the lobbyist cannot force the legislators to stop at some $\ell < \frac{1}{\alpha} < 1$ and at the same time find a persuasive intermediary given this short duration. As a result, the lobbyist cannot exert any influence on the policy choice so the choice of intermediary and duration is irrelevant.

Effect of k. Finally, we illustrate by means of a numerical example that the problem is more subtle when the legislators face an exogenous cost of waiting in the model above: in this case the internal investigation still pressures the lobbyist to shorten his investigation, but it can allow him to choose a more friendly intermediary. Intuitively, the lobbyist is still concerned that running a longer investigation increases the chances that the legislature discovers the truth in the meantime. On the other hand, the fact that the legislature can now obtain its own information increases its benefit of waiting, which relaxes the constraint faced by the lobbyist to persuade the legislators to wait.

Suppose that legislators face a positive $k > 0$ but the lobbyist still faces no cost other than the endogenous pressure from the internal investigation. Consider the case where $\hat{x}_M = \frac{3}{4}$ and $k \in \left(0, \frac{1}{4}\right)$.

Proposition 9. *Let* $\hat{x}_M = \frac{3}{4}$ *and* $k < \frac{1}{4}$.

- *If* $\alpha = 0$, *then the lobbyist's equilibrium is* $(\ell^*, \hat{x}_j^*) = (1, \frac{1}{2} + k)$
- *If* $\alpha = 1$ *then the lobbyist's equilibrium strategy is* $\ell^* = 2\sqrt{\frac{2}{3}} - 1 < 1$ *and* $\hat{x}_j^* = \frac{1}{2}\sqrt{\frac{3}{2}}$. *This intermediary is more moderate,* $\hat{x}_j^* > \frac{1}{2} + k$, *if only if* $k < \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$.

Proof of Proposition 9. Suppose first that $\alpha = 0$ so the legislature cannot acquire information. In this case, the lobbyist solves $\max_{x_j,\ell} \quad 1 - \hat{x}_j$ s.t $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\ell(1-\hat{x}_M)+2\hat{x}_M-1}$ and $\ell \in \left[\frac{(1-\hat{x}_M)(2\hat{x}_M-1)}{2\hat{x}_M(1-\hat{x}_M)-k}, 1\right]$. The resulting equilibrium length of investigation is $\ell^* = 1$ and the intermediary is $\hat{x}_j^* = 2\hat{x}_M - 1 + k = \frac{1}{2} + k$.

Suppose now that $\alpha = 1$. The median's expected payoff at time ℓ equals

$$V^M_{\ell,x_j} = \hat{x}_j\left[\ell + (1-\ell)\hat{x}_M\right] + (1-\hat{x}_j)\left[\ell\left(\frac{1-\hat{x}_M}{1-\hat{x}_j}\right) + (1-\ell)(1-\hat{x}_M)\right].$$

Given a certain $(\ell, \hat{x}_j)$, the median best responds with a length of investigation $t(\ell, \hat{x}_j)$ that solves: $\frac{dU_M(t)}{dt} = 1 - V^M_{\ell,x_j} + k\ell - 2kt = 0 \Leftrightarrow t^*(\ell, \hat{x}_j) = \frac{1}{2k}\left[1 - V^M_{\hat{x}_j,\ell} + k\ell\right]$. If this value is larger than ℓ, then $t^*(\ell, \hat{x}_j) = \ell$. The lobbyist then chooses ℓ and $\hat{x}_j$ to maximize his payoff given this best-response. If $t^*(\ell, \hat{x}_j) < \ell$, then the lobbyist solves

$$\max_{\ell,\hat{x}_j} t^*(\hat{x}_j, \ell)(1 - \hat{x}_M) + (1 - t^*(\hat{x}_j, \ell))(1 - \hat{x}_j),$$

subject to the constraints that (1) the median at time $t(\hat{x}_j, \ell)$ is willing to wait until time ℓ if she did not obtain any internal information and (2) there exists an intermediary $\hat{x}_j < \hat{x}_M$ that can persuade the median. Solving this optimization problem gives $(l^*, \hat{x}^*_j) = (1, \frac{3-2k}{4})$. We can then verify that the payoff from this strategy is greater than the payoff from inducing an interior t^* for all $k \in (0, \frac{1}{4})$. □

B.4 Extension: Competing lobbies

In this extension, we consider the following model. There are two lobbyists: L_0 and L_1. L_0 only gains when policy $x = 0$ is selected and L_1 only gains when policy $x = 1$ is selected (like the single lobby in our baseline model). At time $t = 0$, each lobby $i \in \{L_0, L_1\}$ simultaneously commits to a duration of investigation ℓ_i and an intermediary $\hat{x}^i_j$ with whom to share the information. When a lobbyist's investigation ends, he observes privately a signal realization $s_i \in [0, 1]$. Each signal is distributed as in our baseline model and independent of the other lobbyist's signal (except when both equal the true state). If a lobby moves before the other, the other lobbyist can observe any previous endorsement.

The legislators' utility functions are the same as in the baseline model with a cost of waiting k. However, we modify the lobbyists' utility functions so that they no longer face a cost of waiting: $U_{L_0}(t) = 1 - x$ and $U_{L_1}(t) = x$ for any t at which the process stops.

Finally, we assume that neither lobby is favored ex-ante, so that $\hat{x}_M = \frac{1}{2}$ and the median chooses each policy with probability $\frac{1}{2}$ when indifferent, and that the median benefits from obtaining information: $k < \frac{1}{2}$.

Single lobby case: suppose that L_1 is the only lobby providing information. Given that the lobbyist faces no cost of waiting, the optimal strategy is to choose $\ell^*_1 = 1$ and the lowest $\hat{x}_j$ such that $\hat{x}_j \geq \frac{k\ell^*_1 + 2\hat{x}_M - 1}{2\ell^*_1(1-\hat{x}_M) + 2\hat{x}_M - 1}$ (as per Lemma 2). Substituting $\ell^*_1 = 1$ and $\hat{x}_M = \frac{1}{2}$ gives $\hat{x}^{1*}_j = k$ for any $k < \frac{1}{2}$. This strategy results in an equilibrium payoff of $U_{L_1} = 1 - k$ and $U_{L_0} = k$.

1. **The competing lobby would benefit from preempting the existing lobby.** We first show that competition can create pressure akin to time pressure because of the threat that the competing lobby may provide information earlier on and induce the legislature to stop before the existing lobby can share information.

   With only L_1 offering information, the competing lobby (L_0) gets a payoff of $U_{L_0} = k$. Holding L_1's strategy constant ($\ell^*_1 = 1$, $\hat{x}^{1*}_j = k$), suppose that L_0 offers to run an investigation until some time $t = \ell_0 < 1$ and to share the results with the median directly: $\hat{x}^0_j = \hat{x}_M$. If the median stops the process at that point, lobby L_0 would win whenever

$s_0 \leq \hat{x}_M$ so his payoff would be $U_{L_0} = \frac{1}{2} > k$.

If ℓ_0 is sufficiently large, the median receives enough information from L_0 that it is not worth waiting for the garbled information from L_1 at $t = 1$. Indeed, if $\ell_0 \geq k$, the median would prefer to stop at $t = \ell_0$ following any realization of s_0. If $s_0 > \hat{x}_M$, the median knows that L_1 would not disclose any s_1 since he is guaranteed a win by remaining silent. If $s_0 < \hat{x}_M$, the median anticipates that it would only possibly change her policy choice when $s_1 > \hat{x}_j^{1*} = k$. However, given $\ell_0 > k$, the median's posterior belief $\mathbb{P}(\omega > \hat{x}_M | s_0 < \hat{x}_M, s_1 \geq k)$ is less than half. So the median anticipates that she will never change her policy decision and therefore prefers to stop immediately.

2. **Competition can lead to moderation.** The threat of preemption described above would force the existing lobby to react to the entry of a competing lobby. We now show that there is an equilibrium of the game with two lobbies that features moderation relative to the equilibrium with a unique lobby.

Proposition 10. *The strategy profile* $((\ell_1, \hat{x}_j^1), (\ell_0, \hat{x}_j^0) = ((1, \hat{x}_M), (1, \hat{x}_M))$ *and a majority of legislators voting to continue until* $t = 1$ *is an equilibrium of this game.*

Proof of Proposition 10. In this equilibrium, the expected payoff of each lobbyist is $U_i = \frac{1}{2}$. Since the game is symmetric, it suffices to show that one lobby has no incentives to deviate. Consider lobby L_1.

(a) No deviation to $\hat{x}_j \neq \hat{x}_M$ at $\ell_1 = 1$: given $t = 1$, the median would learn the true state from the other lobby and ignore the signal of the lobbyist who deviated, so this cannot be profitable.

(b) **Claim:** Any possibly profitable deviation to $\ell_1' < 1$ could only possibly be profitable if accompanied by a deviation to $\hat{x}_j < \hat{x}_M$.

Proof: Suppose by contradiction that L_1 deviates to $\ell_1' < 1$ and $\hat{x}_M$. If $s_1 < \hat{x}_M$, then the median stops immediately as she knows that the next lobbyist would conceal any evidence he obtains to secure his win. This happens with probability $\frac{1}{2}$, so the deviating lobbyist wins with probability at most $\frac{1}{2}$. Therefore, this cannot be a profitable deviation.

(c) No deviation to $\ell_1' < 1$ and $\hat{x}_j < \hat{x}_M$. First note that, the median would stop following an endorsement $m_j = 0$ as it knows that the next lobbyist would remain silent. Next, we show that the median always waits for the second signal following an endorsement in favor of the deviating lobbyist: $m_j = 1$. If the median continues, she receives perfect information from the second lobbyist, so her continuation value is $U_M = 1 - k(1 - \ell_1')$. If she stops, she chooses $x = 1$ as long as ℓ_1' is large enough (otherwise, she never chooses $x = 1$ so this cannot be a profitable deviation). Her expected payoff is then $U_M = \ell_1' \times \frac{1-\hat{x}_M}{1-\hat{x}_j} + (1 - \ell_1')(1 - \hat{x}_M)$. Finally, note that $U_M = 1 - k(1 - \ell_1') > \ell_1' \times \frac{1-\hat{x}_M}{1-\hat{x}_j} + (1 - \ell_1')(1 - \hat{x}_M)$ since $1 - k > \ell_1' \left(\frac{1}{2(1-\hat{x}_j)} - \frac{1}{2} - k \right) + \frac{1}{2} \Leftrightarrow \frac{1}{2} - k > \frac{1}{2} \ell_1' \left(\frac{\hat{x}_j}{1-\hat{x}_j} - 2k \right)$ for any $k, \hat{x}_j < \frac{1}{2}$ and any $\ell_1' < 1$.

$\square$

B.5 Investing resources to investigate

We generalize the baseline by assuming that the signal is equal to the state at time t with probability $\alpha \times t$ for $t \in \left[0, \frac{1}{\alpha}\right]$ for some $\alpha \geq 1$ (in our baseline model, $\alpha = 1$). For $t \geq \frac{1}{\alpha}$, the investigation perfectly reveals the state ω. In addition, suppose the lobbyist can invest in accelerating information generation: he can choose $\alpha \in [1, +\infty)$ at cost $c \times \alpha$. Finally, we assume that this is the only cost faced by the lobbyist: the cost of waiting is 0 for the lobbyist, but remains $k > 0$ for the legislators.

A higher speed of learning ($\alpha > 1$) increases the legislator's benefit of waiting for some time t but not the cost of waiting. The median's waiting constraint thus becomes: $\hat{x}_j \geq \frac{k\ell+2\hat{x}_M-1}{2\alpha\ell(1-\hat{x}_M)+2\hat{x}_M-1}$.The lobbyist's problem is then

$$\max_{\alpha, x_j, \ell} \quad 1 - \hat{x}_j \quad \text{s.t. } \hat{x}_j \geq \frac{k\ell + 2\hat{x}_M - 1}{2\alpha\ell(1-\hat{x}_M) + 2\hat{x}_M - 1}, \quad \ell \in \left[\frac{(1-\hat{x}_)(2\hat{x}_M - 1)}{2\alpha\hat{x}_M(1-\hat{x}_M) - k}, \frac{1}{\alpha}\right]$$

Solving this problem gives: $\ell^\alpha = \begin{cases} \sqrt{\frac{c}{k}} & \text{if } c < k \\ 1 & \text{if } c \geq k \end{cases}$, $\alpha^* = \begin{cases} \sqrt{\frac{k}{c}} & \text{if } c < k \\ 1 & \text{if } c \geq k \end{cases}$

and $\hat{x}_j^\alpha = \begin{cases} 2\hat{x}_M - 1 + \sqrt{ck} & \text{if } c < k \\ 2\hat{x}_M - 1 + k & \text{if } c \geq k \end{cases}$.

Since $\sqrt{ck} > k$ whenever $c < k$ and since the friendliest intermediary when we impose an exogenous speed of investigation $\alpha = 1$ is $\hat{x}_j = 2\hat{x}_M - 1 + k$, the lobbyist is always able to choose a weakly friendlier intermediary when he can invest in speeding up the investigation.

B.6 Allowing the intermediary to share evidence

In our model, we restrict the intermediary to making a cheap talk endorsement of one of the policies. However, if the intermediary could also share the evidence obtained from the lobbyist, she would prefer not to do it, or would do it in a way that is outcome-equivalent to the binary endorsement. In equilibrium, the intermediary always obtains her preferred policy as the legislature follows her endorsement. One worry when the intermediary can disclose the lobbyist's evidence is that unraveling may occur other legislators might form unfavorable beliefs about evidence that is concealed, thus forcing the intermediary to disclose that evidence. However, because of the threshold structure of the intermediary's preferences, unraveling does not necessarily occur. Indeed, unraveling would occur if disclosing a marginally higher signal than the threshold below or above which the intermediary conceals evidence increases the intermediary's payoff. This is not the case here.

However, it is possible to construct equilibria in which the intermediary is forced to disclose any evidence. In these equilibria, the lobbyist can no longer rely on private targeting of intermediaries.

We illustrate this with an equilibrium in which the intermediary does disclose some of the evidence shared by the lobbyist, yet the lobbyist's strategy and the outcome is the same as in our baseline model.[13]

[13]There are also simpler equilibria in which the intermediary *never* discloses any evidence and instead makes

Consider the following modification to our original model. In addition to an endorsement $m_j \in \{0,1\}$, the intermediary can also choose whether to share the evidence s provided by the lobbyist, denoted by $\sigma(s) = s$ or to conceal it, denoted by $\sigma(s) = \emptyset$. If the intermediary discloses it, it becomes publicly available to all the other legislators. As before, the intermediary makes that decision at $t = \ell^*$. If she decides to share the evidence, the other legislator's beliefs are completely determined by Bayes rule. If she decides to conceal the evidence, however, the other legislators' beliefs depend on the intermediary's strategy.

Proposition 11. *The following strategies and beliefs constitute an equilibrium of the modified game:*

1. *The lobbyist chooses $\ell = \ell^*(k)$ and $\hat{x}_j^*(k)$, where $\ell^*(k)$ and $\hat{x}_j^*(k)$ are the equilibrium duration and intermediary in the original game as defined in Proposition 1.*

2. *The lobbyist discloses any $s \in [0,1]$ to the intermediary.*

3. *If the lobbyist shares evidence s, the intermediary discloses the evidence and endorses policy $x = 0$ if $s < \hat{x}_j^*$ but conceals the evidence and endorses policy $x = 1$ if $s \geq \hat{x}_j^*$:*

$$\sigma_j^*(s) = \begin{cases} s & \text{if } s < \hat{x}_j \\ \emptyset & \text{if } s \geq \hat{x}_j \end{cases}, \quad m_j^*(s) = \begin{cases} 0 & \text{if } s < \hat{x}_j \\ 1 & \text{if } s \geq \hat{x}_j \end{cases}$$

   If the lobbyist shares no evidence, the intermediary shares no evidence but endorses publicly policy $x = 0$: $\sigma_j^*(\emptyset) = \emptyset$ and $m_j^*(\emptyset) = 0$.

4. *A majority of legislators wait until $t = \ell^*$.*

5. *Legislators with $\hat{x}_i \in [\underline{x}, \bar{x}]$ vote $x = 1$ if and only if $\sigma_{j^*} = \emptyset$ and $m_j^* = 1$. Legislators with $\hat{x}_i > \bar{x}$ vote $x = 0$ for any σ_{j^*} and any m_{j^*} and those with $\hat{x}_i < \underline{x}$ vote $x = 1$ for any σ_{j^*} and any m_{j^*}. Where $\underline{x} < \hat{x}_M < \bar{x}$ are two thresholds.*

6. *Legislators form the following beliefs given m_j:*

   (a) *If $\sigma_{j^*} = z$, $\mathbb{P}(s = z | \sigma_{j^*}, m_j = 1) = 1$.*

   (b) *If $\sigma_{j^*} = \emptyset$ and $m_{j^*} = 1$, $\mathbb{P}(s < \hat{x}_i | \sigma_{j^*}, m_{j^*}) = \mathbb{P}(s < \hat{x}_i | s \geq \hat{x}_j^*)$ for any $\hat{x}_i$.*

   (c) *If $\sigma_{j^*} = \emptyset$ and $m_{j^*} = 0$, $\mathbb{P}(s < \hat{x}_i | \sigma_{j^*}, m_{j^*}) = \mathbb{P}(s < \hat{x}_i)$ for any $\hat{x}_i$.*

Proof of Proposition 11. Given the intermediary's strategy, the lobbyist's choice of duration and intermediary is optimal for the same reasons as in Proposition 1. Given their beliefs, the legislators' procedural and policy votes are also optimal for the same reasons as in Proposition 1. We therefore only need to check that (1) the intermediary has no incentives to deviate from her strategy, (2) the lobbyist has no incentives to deviate from his disclosure strategy, and (3) the legislator's beliefs on the equilibrium path are consistent with the intermediary and the lobbyist's strategies.

endorsements as in the original model. We prove the existence of a different equilibrium to illustrate that it is possible to recover our characterization even when the intermediary does share some evidence.

1. **Intermediary strategy:** In equilibrium, the intermediary induces policy $x = 1$ whenever $\mathbb{P}(\omega \geq \hat{x}_j^*|s) \geq \frac{1}{2}$ and policy $x = 0$ to be chosen whenever $\mathbb{P}(\omega < \hat{x}_j^*|s) \geq \frac{1}{2}$. Suppose that she observes $s < \hat{x}_j^*$, then deviating to $\sigma_j = \emptyset$ and $m_j = 1$ would induce $x = 1$. However, given $s < \hat{x}_j^*$, the intermediary would prefer $x = 0$, so this is not a profitable deviation. If $s \in [\hat{x}_j^*, \hat{x}_M)$, then deviating to $\sigma_j = s$ or to $\sigma_j = \emptyset$ but $m_j = 0$ would induce $x = 0$ while the intermediary prefers $x = 1$. Finally, if $s \geq \hat{x}_M$, deviating to $\sigma_j = s$ would have no impact on the policy chosen, while deviating to $\sigma_j = \emptyset$ but $m_j = 0$ would induce $x = 0$ while the intermediary prefers $x = 1$. Following a deviation by the lobbyist to disclosing no evidence, the intermediary can form any beliefs. In particular, believing that $s < \hat{x}_j^*$ is consistent with the equilibrium concept, in which case recommending $m_j = 0$ to induce $x = 0$ is indeed optimal.

2. **Lobbyist disclosure:** if the lobbyist deviates to concealing some signal s rather than disclosing it, the intermediary will share endorsement $m_j = 0$ which induces $x = 0$. This either leaves the lobbyist's utility unchanged or strictly reduces it.

3. **Legislators' beliefs:** Given the intermediary's strategy, the legislator's beliefs are consistent with Bayes rule **on the equilibrium path**. If $\sigma_{j^*} = z$, then the beliefs are fully determined by the information structure as information is hard evidence and public. If $\sigma_{j^*} = \emptyset$ and $m_{j^*} = 1$, the legislators infer that the intermediary observed $s \geq \hat{x}_j^*$. **Off-path,** if the lobbyist discloses no evidence, then the intermediary shares $\sigma_{j^*} = \emptyset$ and $m_{j^*} = 0$, the legislators infer that the intermediary observed $s = \emptyset$ and can form any beliefs since this is off-path.

□

Equilibria with full disclosure. We conclude by sketching how an equilibrium where all information is disclosed can be sustained. Consider an equilibrium in which any legislator discloses any signal s shared by the lobbyist if they are selected as an intermediary, and the lobbyist always discloses any signal s that it obtains to the intermediary. If the intermediary conceals some signal, the other legislators can form off-equilibrium beliefs that the signal observed was some $s \leq \hat{x}_M$ so a majority votes for $x = 0$. The intermediary thus never has a strict incentive to conceal information. Similarly, if the lobbyist conceals information, the intermediary is forced to report no evidence, which also leads to policy $x = 0$. As a result, neither the lobbyist nor the intermediary have incentives to deviate to ever concealing information. The lobbyist has no incentives to select a different intermediary as they all lead to the same distribution over outcomes. This equilibrium is always equivalent to engaging in public lobbying, so there is no gain from private lobbying.